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Application of interval-valued T-spherical fuzzy Dombi Hamy mean operators in the antiviral mask selection against COVID-19

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Abstract

This study introduces Interval-valued T-Spherical Fuzzy Dombi Hamy Mean (IVT-SPFDHM) Operators as a powerful tool for group decision-making. The IVT-SPFDHM operators allow for prioritization of fuzzy data, effectively managing uncertainty. Its framework is applied in diverse group decision-making contexts, presenting its adaptability and robustness in addressing complex real-world problems. This study examines the Multi-Attribute Decision-Making (MADM) issue in the IVT-SPFDHM environment where the qualifications and expertise are at varying levels of necessity. We regard the novel Aczel-Alsina aggregation operators (AOs) as the most recently created AOs, capable of handling considerable uncertainty. To propose some AOs, we investigated the Hamy mean (HM) operator in the following environment: Interval-valued T-Spherical Fuzzy weighted Dombi Hamy mean (IVT-SPWDHM) operator, stretch esteemed interval valued T-Spherical Dual Dombi Hamy Mean (IVT-SPFDDHM), and Interval-valued T-Spherical Fuzzy weighted Dual Dombi Hamy Mean (IVT-SPFWDDHM). The weights for prioritization are derived from the knowledge of experts, and the proposed operators can capture the phenomenon of prioritization among the aggregated arguments. The MADM models are then planned using the IVPWDHM and IVPWDDHM operators. Finally, we provided a sample example within the prioritized ones to select the best antiviral mask for fighting COVID.

Keywords: IVT-SPFDHM, IVT-SPFDDHM, IVT-SPFWDHM, IVT-SPFWDDHM, Multi-Attribute Decision-Making (MADM).

1. Introduction

The Multi-Attribute Decision Making (MADM) in fuzzy environments is a specialized area of research and practice within the broader field of decision-making science. It deals with complex decision-making scenarios where a group of decision-makers collaboratively assess and select the best alternative among several options while considering multiple criteria and the information is inherently uncertain. The MADM in fuzzy environments aims to address the challenges of aggregating diverse preferences, handling imprecise data, and reconciling conflicting criteria. It offers a systematic approach to reach a consensus that reflects the collective wisdom of the group, while accommodating fuzziness and uncertainty.

Fuzzy set (FS) theory is a mathematical framework that extends classical set theory by introducing the concept of "FS," which allows for the presentation of degrees of membership or uncertainty. Unlike classical sets, where an element either belongs to a set (with a membership degree of 1) or does not (with a membership degree of 0), a FS assigns membership degrees between 0 and 1, reflecting the partial or gradual membership of elements in a set. This approach is especially useful for modelling and handling uncertainty and vagueness in various real-world applications. FS theory provides a powerful framework for addressing complex problems that involve uncertainty and ambiguity. By allowing for the presentation of partial membership and gradual transition between categories, it has become an indispensable tool in a wide range of applications where traditional binary logic falls short. FS theory, developed by Zadeh (1965), has found widespread applications in diverse fields such as artificial intelligence, decision-making, control systems, pattern recognition, and linguistics. It provides a flexible framework for capturing human reasoning and dealing with imprecise information, making it an essential tool in modern problem-solving which is a function that presents the idea of membership element in the interval form $[0,1]$. Zadeh (1965) defined the operation of Fuzzy Set (FS). Atanassov (1999) defined the interval-valued intuitionistic fuzzy sets (IVIFSs) based on the membership grade (MG) and non-membership grade (NG) and this theory increased the concept of the FS. Atanassov (1999) defined properties of the interval-valued intuitionistic fuzzy sets (IVIFSs). Yager (2008) developed the idea of Pythagorean Fuzzy set (PyFs) based on the MG and NG interval of [0,1]. Yager (2017) further increased the rank of MG and NG and introduced the q-Rung Orthopair Fuzzy Set (q-ROFS), the both using frame work of IFSs and PyFSs.

Real-world decision-making difficulties have been successfully solved by IFSs, PyFSs, and q-ROFSs; nevertheless, in some cases, it is necessary to abstain or refuse rather than fully commit to a yes-or-no response. IFSs, PyFSs, and q-ROFSs are unable to produce satisfactory results under these circumstances. Cuong et al. (2016) considered the overall concept of FS and IFS and introduced the Picture Fuzzy Set (PFS) with an additional degree called abstinence degree (AD). The MD, abstinence degree, and NMD constitute the PFS, which is seen as an extension of the FS and IFS with the restriction that their aggregate must be less than or equal to one. For instance, when a PFS evaluation of an object is provided by an expert, the abstinence degree is involved, which prevents the IFS, PyFS, and q-ROFS from handling the data. The PFS is a broader version of the sets that were previously mentioned. The correlation coefficient under PF information was defined by Singh (2015). A few PF clustering-based forecasting techniques were presented by Son and Thong (2017) for weather forecasting using satellite picture sequences. Developing the fuzzy and IF databases Dinh et al. (2015) suggest the picture database. PFSs are restricted in the same way as IFSs, meaning that the total of MD, AD, and NMD must equal one or be less than one. Mahmood et al. (2019) created the framework of a spherical fuzzy set (SFS) by extending the PFS's range in order to overcome this restriction. The T-spherical fuzzy set (T-SFS) is a further extension of the SFS notion. The constraints were updated to include the parameter "q." This parameter's value can be determined based on the circumstances. The T-SFS lowers to the PFS if the parameter value is one or two or SFS, if appropriate.

1.1 Literature review

Numerous researchers have examined the uses of T-SFS and SFS since the publications of Mahmood et al. (2019). Ullah et al. (2018), for instance, proposed a number of unique similarity measures, such as set theoretic similarity measures for SFS and T-SFSs, cosine similarity measurements and grey similarity measures. A few proposed clustering algorithms for decision-making problems and correlation coefficients for T-SFSs are found in Ullah et al. (2020). Ullah et al. (2018) propose an extended T-SFS known as an interval-valued T-SFS. In order to assess investment policy and pattern recognition issues, they defined a few similarity metrics and aggregation operators for interval-valued T-SFSs. A probability-based interactive averaging aggregation operator for T-SFSs was developed by Zeng et al. (2019). Nine similarity metrics were derived using the T-SFS by Wei et al. (2019). Shortest path issues and medical diagnosis were addressed by Jin et al. (2020) using the interval-valued T-SFS. In the context of the T-SFS structure, Wu et al. (2019) explored the shortcomings of the divergence measures that are currently in use and suggested new ones. The T-SFS Hamacher t-norm and t-conorm aggregation operators were first presented by Jin et al. (2021). According to Munir et al. (2020), Einstein aggregation operators were introduced, along with some enhanced algebraic operations. In order to address the MADM issues, Garg et al. (2018) created a DM methodology and defined a number of new, enhanced aggregation operators for T-SFSs. Wu et al. (2019) examined T-SFS divergence measures and provided an application for pattern identification. Liao et al. (2021) provided the new technical face mask wearing idea for fighting COVID. Pemmada et al. (2020) expand the viral properties of COVID based on antiviral coatings. O'Dowd et al. (2020) provided the concept of current materials for COVID face masks. Pradhan et al. (2020) expand the current COVID interventions. Gope et al. (2020) provided the concept for mask material used during COVID. Mongia et al. (2021) expand the concept of which drugs are used during COVID.

1.2 Motivation for the research

Experts can model ambiguous and uncertain information using the T-SFS's numerical framework. These studies are capable of dealing with information that can not only be quantified and is uncertain. But many real-world decision-making scenarios have qualitative elements that communicate imprecision and ambiguity. For example, when assessing a motor bike's price, professionals can communicate their evaluation information using words like "very cheap," "low," "medium," "high," and "extremely expensive." The linguistic term set (LTS), a fuzzy linguistic technique, was proposed by Zadeh (1983) to model these qualitative evaluations. This has been a hot research topic and received a lot of attention in the last few years. Researchers have developed numerous extensions of the LTS, which are documented in the literature. Chen et al. (2015) suggest the linguistic IFS (LIFS), a hybrid of the IFS and LTS. Extensions of the LIFS, such as linguistic PYFS (LPYFS) (Garg, 2018), linguistic ROFS (L q-ROFS) (Lin et al., 2020; Gurmani et al., 2021), linguistic PFS (LPFS) (Liu et al., 2020), and linguistic SFS (LSFS) (Jin et al., 2019), have been proposed for some situations where the LIFS fails. Assume we have a continuous LTS $S = \{st| \ t \in [0, 1]\}$ and LPFS (3, 2, 5). For example, in an actual decision-making scenario, if an expert offers an assessment in the form of LPFS as (s^3, $s^2 s^5$) and t = 9, then by the LPFS condition, $3 + 2 + 5 > 9$. In this case, the restriction condition causes the LPFS to fail. To handle such information, the LSFS seems to be a helpful tool; that is, $(3)2 + (2)2 + (5)2 < (9)2$. It is evident that LSFSs are more versatile than LPFSs. Unfortunately, due to limitations, neither the LPFS nor the LSFS can handle expert opinions presented in the form of (s6, s5, s7) when they occur in real-world problems. In order to give experts more flexibility and prevent data loss, we expanded the idea of the LSFS and proposed the idea of a linguistic interval-valued T-spherical fuzzy set (LIVTSFS).

1.3 Aim of the Study

Moreover, MADM is a rapidly growing field that has been extensively investigated by using weighted average, weighted geometric operators, similarity and distance measures and some other well-known traditional decision-making techniques, such as the VIKOR method (Opricović, 1998), TODIM method (Gomes and Lima, 1991), MABAC method (Pamučar and Ćirović, 2015) and crisp TOPSIS method (Hwang and Youn, 1981). The Entropy Weight Method is proposed for MADM problems in which the assessment of attributes can include uncertainty (Zhou et al., 2020a; Yeh, 2002; Bao et al. 2017; Heidary et al., 2021; Zhou et al., 2020b). Evidential reasoning provides an efficient process for dealing with MADM problems. Zhou et al. (2019) added fixed interval attributes and deviated intervals to the conventional evidential reasoning method. Our work is an adaptation of the current TOPSIS methodology for the LIVT-SFS setting. The TOPSIS technique is a straightforward and efficient method for making decisions. It looks for the option that is closest to both positive and negative ideal solutions in terms of distance.

Numerous academics have proposed TOPSIS extensions for different fuzzy settings in the last few decades, including Behzadian et al. (2012).

1.4 Contributions

Thus, the primary objective of this article is to suggest enhanced Dombi operational laws, enhance current Dombi AOs, and create some new AOs. These include the weighted form of the T-spherical fuzzy Dombi prioritized Fuzzy (T-SFDPF) operator and its application to MAGDM, as well as the weighted form of the T-spherical fuzzy dual Dombi prioritized Fuzzy (T-SFDDPF) operator. Several fundamental properties of these recently proposed AOs are also covered. The following is a list of paper's contributions:

- The IVT-SPDHM, IVT-SPDDHM, IVT-SPWDHM, and IVT-SPWDDHM operators are some of the new AOs we suggested for the IVPFs, and some relevant attributes are addressed.
  Based on the IVT-SPDHM or IVT-SPDDHM operator, we created a brand-new IVT-SFs MADM approach.
- By addressing investment decision-related problems, we evaluated the applicability of our suggested aggregation function-based MADM method.
- To contrast the outcomes attained with other current AOs and the suggested prioritized AOs of T-SFs.

1.5 Organization of the study

This document has the following structure. We covered certain background information on fuzzy frameworks and aggregation operations in the Section 1. The section 2 covered fundamental terminology pertaining to T-SFSs and IVT-SPFSs. In the section 3 are examined Dombi operations within the context of IVT-SFs based on IVT-SPFDW and IVT-SPFDDW operators. In the Section 4 we presented an example for the selection of antiviral mask. In the section 5 of the article it is done the summary with basic comments.

2. Preliminaries

2.1 Interval-valued T-spherical Fuzzy Prioritization

The fundamental ideas of IVSFs and IVT-SPFs are presented in this section as an introduction to the suggested work. These ideas will help our comprehension of this content.

Definition 1. (Khalil et al., 2019): Let X be a vast expanse of discussion, an IVT-SFs Q' over X is an item having the structure as follows:

$$Q' = \left\{\left(x, {\theta^n}_{Q'}(x), {\beta^n}_{Q'}(x), {\alpha^n}_{Q'}(x)\right) \ |x \in X\right\}$$

Where ${\theta^n}_{Q'}(x) \subseteq [0,1]$, ${\beta^n}_{Q'}(x) \subseteq [0,1]$ and ${\alpha^n}_{Q'}(x)$ are interval numbers $0 \leq \sup\left({\theta^n}_{Q'}(x)\right) + sup\left({\beta^n}_{Q'}(x)\right) + sup\left({\alpha^n}_{Q'}(x)\ \right) \leq 1, \forall\, x \in X$. For convenience, let ${\theta^n}_{Q'}(x) = ([\alpha^n, \beta^n])$,${\beta^n}_{Q'}(x) = ([\ \gamma^n, \delta^n])$ and ${\alpha^n}_{Q'}(x) = ([\ \varrho^n, \vartheta^n])$ so $\alpha' = ([\alpha^n, \beta^n\ ], [\gamma^n, \delta^n], [\ \varrho^n, \vartheta^n])$ is IVPFNs**.**

Definition 2. (Xu and Yager, 2008): Let $\alpha' = ([\alpha^n, \beta^n\ ], [\gamma^n, \delta^n], [\ \varrho^n, \vartheta^n])$ be an IVT-SPFN, a score capability S can be characterized as follows:

$$S(\alpha) = \frac{1}{3}(2 + \alpha^n - \beta^n + \gamma^n - \delta^n + \varrho^n - \vartheta^n), S(\alpha') \in [-1, 1] \quad (1)$$

Definition 3. (Xu and Yager, 2008): Let $\alpha' = ([\alpha^n, \beta^n\ ], [\gamma^n, \delta^n], [\ \varrho^n, \vartheta^n])$ be an IVT-SPFN, a score capability H can be characterized as

$$H(\alpha) = \frac{1}{3}(2 + \alpha^n + \beta^n + \gamma^n + \delta^n + \varrho^n + \vartheta^n), H(\alpha') \in [0, 1] \quad (2)$$

To assess the degree of accuracy of the $IVT - SPFN\ \alpha' = ([\alpha^n, \beta^n\ ], [\gamma^n, \delta^n], [\ \varrho^n, \vartheta^n])$.

Definition 4. (Xu and Yager, 2008): $\alpha_{\mathbf{1}} = ([\alpha_1^n, \beta_1^n], [\gamma_1^n, \delta_1^n], [\varrho_1^n, \vartheta_1^n])$ and $\alpha_{\mathbf{2}} = ([\alpha_2^n, \beta_2^n], [\gamma_2^n, \delta_2^n], [\varrho_2^n, \vartheta_2^n])$ be three IVT-SFNs, $S(\alpha_1) = \frac{1}{3}(2 + \alpha_1^n - \beta_1^n + \gamma_1^n - \delta_1^n + \varrho_1^n - \vartheta_1^n)$ and $S(\alpha_2) = \frac{1}{3}(2 + \alpha_2^n - \beta_2^n + \gamma_2^n - \delta_2^n + \varrho_2^n - \vartheta_2^n)$ be the scores of φ'_1 and φ'_2 individually, and let $H(\alpha_1) = \frac{1}{3}(2 + \alpha_1^n + \beta_1^n + \gamma_1^n + \delta_1^n + \varrho_1^n + \vartheta_1^n)$and $H(\alpha_2) = \frac{1}{3}(2 + \alpha_2^n + \beta_2^n + \gamma_2^n + \delta_2^n + \varrho_2^n + \vartheta_2^n)$ be the accuracy degrees of α_1 and α_2, respectively, then if $S(\alpha_1) < S(\alpha_2)$ then $(\alpha_1) < (\alpha'_2)$; if $S(\alpha_1) < S(\alpha_2)$ then (1) if $H(\alpha_1) < H(\alpha_2)$ then $(\alpha'_1) < (\alpha'_2)$; if $H(\varphi'_1) < H(\varphi'_2)$ then $(\alpha'_1) < (\alpha'_2)$.

Definition 5. (Xu and Yager, 2008): The two IVT-SFNs $\alpha'_1 = ([\alpha_1^n, \beta_1^n], [\gamma_1^n, \delta_1^n], [\varrho_1^n, \vartheta_1^n])$ and $\alpha'_1 = ([\alpha_2^n, \beta_2^n], [\gamma_2^n, \delta_2^n], [\varrho_2^n, \vartheta_2^n])$ the observing functional regulations are characterized as follows:

1) $\alpha'_1 \oplus \alpha'_2 = \begin{pmatrix} \sqrt[n]{[\alpha_1^n + \alpha_2^n - \alpha_1^n \alpha_2^n, \beta_1^n + \beta_2^n - \beta_1^n \beta_2^n,]}, \\ [\gamma_1^n \gamma_2^n, \delta_1^n \delta_2^n], [\varrho_1^n \varrho_2^n, \vartheta_1^n \vartheta_2^n] \end{pmatrix}$

2) $\alpha'_1 \otimes \alpha'_2 = \begin{pmatrix} \sqrt[n]{[\alpha_1^n \alpha_2^n, \beta_1^n \beta_2^n]} \\ [\gamma_1^n + \gamma_2^n - \gamma_1^n \gamma_2^n, \delta_1^n + \delta_2^n - \delta_1^n \delta_2^n] \\ [\varrho_1^n + \varrho_2^n - \varrho_1^n \varrho_2^n, \vartheta_1^n + \vartheta_2^n - \vartheta_1^n \vartheta_2^n] \end{pmatrix}$

3) $\tau \alpha'_1 = \left(\sqrt[n]{[1 - (1 - \alpha_1^n)^\tau, 1 - (1 - \beta_1^n)^\tau]}, [(\gamma_1^n)^\tau, (\delta_1^n)^\tau], [(\delta_1^n)^\tau, (\vartheta_1^n)^\tau]\right), \tau > 0$

4) $\alpha_1'^{\tau} = \begin{pmatrix} \sqrt[n]{[(\alpha_1^n)^\tau, (\beta_1^n)^\tau]}, [1 - (1 - \gamma_1^n)^\tau, 1 - (1 - \delta_1^n)^\tau], \\ [1 - (1 - \varrho_1^n)^\tau, 1 - (1 - \vartheta_1^n)^\tau] \end{pmatrix}, \tau > 0$

2.2 HM Operator

The HM operator was suggested by Hara et al. (1998), where by using 2-tuple linguistic neutrosophic numbers (2TLNNs) we extend the HM, weighted Hamy mean (WHM), dual Hamy mean (DHM), and weighted dual Hamy mean (WDHM) operators in this study (Wu et al., 2018).

Definition 6. (Dombi, 1982): The HM operator is characterized as follows:

$HM^x \alpha'_1, \alpha'_2, \ldots, \alpha'_{\mathfrak{h}}, = \frac{\sum_{1 \le < \cdots \mathfrak{r}_x \le \mathfrak{h}} \left(\prod_{\ell=1}^{x} \varphi'_{\mathfrak{r}\ell}\right)^{1/x}}{C_{\mathfrak{h}}^x}$

Where xa parameter is $x = 1,2, \ldots, \mathfrak{h}, \mathfrak{r}_1, \mathfrak{r}_2, \ldots, \mathfrak{r}_x$ are x whole number qualities taken from the structure of the set $\{1,2, \ldots, \mathfrak{h}\}$ of ϱ whole number qualities $C_{\mathfrak{h}}^x$ is the binomial coefficient, $C_{\mathfrak{h}}^x = \frac{\mathfrak{h}!}{x!(\mathfrak{h}-x)!}$.

2.3 Dombi Operations of IVT-SPFNs

This section begins with an introduction to the four fundamental operations for IVT-SFNs (Dombi, 1982) and ends with a helpful example and the definition of T-norm and T-conorm DM. Additionally, some unverified results are provided that are easily verifiable.

Definition 7: Dombi (1982) proposed a generator to the created Dombi T-norm and T-conorm which are displayed as follows:

$$D(q, r) = \frac{1}{1 + \left(\left(\frac{1-q}{q}\right)^\beta + \left(\frac{1-r}{r}\right)^\beta\right)^{1/\beta}} \quad (3)$$

$$D^c(q, r) = 1 - \frac{1}{1 + \left(\left(\frac{q}{1-q}\right)^\beta + \left(\frac{r}{1-r}\right)^\beta\right)^{\frac{1}{\beta}}} \quad (4)$$

where $\beta > 0, (q, r) \in [0,1]$. Based on the Dombi T-norm and T-conorm, we are able to provide the IVT-SPFNs functional rules.

Definition 8. The two IVT-SPFNs $\alpha'_1 = \left([\alpha_1^n, \beta_{1\,1}^n], [\gamma_1^n, \delta_1^n], [\varrho_1^n, \vartheta_1^n]\right)$ and $\alpha'_2 = ([\alpha_2^n, \beta_2^n], [\gamma_2^n, \delta_2^n], [\varrho_2^n, \vartheta_2^n])$ $\tau > 0$, the Dombi operational laws are characterized as follows:

$$\varphi'_1 \oplus \varphi'_2 = \left(\begin{array}{c} \left[1-\frac{1}{1+\left(\left(\frac{\alpha_1^n}{1-\alpha_1^n}\right)^{\tau}+\left(\frac{\alpha_2^n}{1-\alpha_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\left(\frac{\beta_1^n}{1-\beta_1^n}\right)^{\tau}+\left(\frac{\beta_2^n}{1-\beta_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\left(\frac{1-\gamma_1^n}{\gamma_1^n}\right)^{\tau}+\left(\frac{1-\gamma_2^n}{\gamma_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\left(\frac{1-\delta_1^n}{\delta_1^n}\right)^{\tau}+\left(\frac{1-\delta_2^n}{\delta_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\left(\frac{1-\varrho_1^n}{\varrho_1^n}\right)^{\tau}+\left(\frac{1-\varrho_2^n}{\varrho_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\left(\frac{1-\vartheta_1^n}{\vartheta_1^n}\right)^{\tau}+\left(\frac{1-\vartheta_2^n}{\vartheta_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \end{array} \right) \tag{5}$$

$$\alpha'_1 \otimes \alpha'_2 = \left(\begin{array}{c} \left[1-\frac{1}{1+\left(\left(\frac{1-\alpha_1^n}{\alpha_1^n}\right)^{\tau}+\left(\frac{1-\alpha_2^n}{\alpha_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\left(\frac{1-\beta_1^n}{\beta_1^n}\right)^{\tau}+\left(\frac{1-\beta_2^n}{\beta_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right], \\ \left[\frac{1}{1+\left(\left(\frac{\gamma_1^n}{1-\gamma_1^n}\right)^{\tau}+\left(\frac{\gamma_2^n}{1-\gamma_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\left(\frac{\delta_1^n}{1-\delta_1^n}\right)^{\tau}+\left(\frac{\delta_2^n}{1-\delta_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\left(\frac{\varrho_1^n}{1-\varrho_1^n}\right)^{\tau}+\left(\frac{\varrho_2^n}{1-\varrho_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\left(\frac{\vartheta_1^n}{1-\vartheta_1^n}\right)^{\tau}+\left(\frac{\vartheta_2^n}{1-\vartheta_2^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \end{array} \right) \tag{6}$$

$$\mathfrak{h}\alpha'_1 = \left(\begin{array}{c} \left[1-\frac{1}{1+\left(\mathfrak{h}\left(\frac{\alpha_1^n}{1-\alpha_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\mathfrak{h}\left(\frac{\beta_1^n}{1-\beta_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ , \left[\frac{1}{1+\left(\mathfrak{h}\left(\frac{1-\gamma_1^n}{\gamma_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\mathfrak{h}\left(\frac{1-\delta_1^n}{\delta_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\mathfrak{h}\left(\frac{1-\varrho_1^n}{\varrho_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\mathfrak{h}\left(\frac{1-\vartheta_1^n}{\vartheta_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \end{array} \right) \tag{7}$$

$$(\alpha'_1)^{\mathfrak{h}} = \left(\begin{array}{c} \left[\frac{1}{1+\left(\mathfrak{h}\left(\frac{1-\alpha_1^n}{\alpha_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\mathfrak{h}\left(\frac{1-\beta_1^n}{\beta_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right], \\ \left[1-\frac{1}{1+\left(\mathfrak{h}\left(\frac{\gamma_1^n}{1-\gamma_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\mathfrak{h}\left(\frac{\delta_1^n}{1-\delta_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(\mathfrak{h}\left(\frac{\varrho_1^n}{1-\varrho_{1\,1}^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\mathfrak{h}\left(\frac{\vartheta_1^n}{1-\vartheta_1^n}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \end{array} \right) \tag{8}$$

3. IVT-SFNs are used by some Dombi Hamy Mean operators

3.1 Dombi Hamy Mean and Dombi Hami Mean Weight operator

This section's objective is to create and research IVT-SPFDHM operators' applicability. Basic features of the aggregation of IVT-SFPDWHM operators are followed by a numerical example.

Definition 9. Let $\alpha'_\ell = \left([\alpha_1^n, \beta_{1\,1}^n], [\gamma_1^n, \delta_1^n], [\varrho_1^n, \vartheta_1^n]\right)$ $(\ell = 1,2,\dots,\mathfrak{h})$ be a set of IVT-SPFNs. The IVT-SPFDHM operator is

$$IVT-SPFDHM^{(x)}\left(\varphi'_1, \varphi'_2, \dots, \varphi'_{\mathfrak{h}}\right) = \frac{\oplus_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\overset{x}{\underset{\ell=1}{\otimes}} {\varphi'_{\mathfrak{r}_\ell}}^{n}\right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^{x}} \tag{9}$$

Theorem 1: Let $\alpha'_\ell = ([\alpha_\ell^n, \beta_\ell^n], [\gamma_\ell^n, \delta_\ell^n], [\varrho_\ell^n, \vartheta_\ell^n])(\ell = 1,2,\dots,\mathfrak{h})$ be a set of IVT-SFNs. The IVT-SPFDHM operator is also the IVT-SPFN where

$$IVT-SPFDHM^{(x)}\left(\varphi'_1, \varphi'_2, \dots, \varphi'_{\mathfrak{h}}\right) = \frac{\oplus_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\overset{x}{\underset{\ell=1}{\otimes}} {\varphi'_{\mathfrak{r}_\ell}}^{n}\right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^{x}} \tag{10}$$

$$= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x} \left(\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}}, 1 - \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x} \left(\frac{1-\beta_{\mathfrak{r}_\ell}^{n}}{\beta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}} \right] \\ \left[\frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x} \left(\frac{\gamma_{\mathfrak{r}_\ell}^{n}}{1-\gamma_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x} \left(\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}} \right] \\ \left[\frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x} \left(\frac{\varrho_{\mathfrak{r}_\ell}^{n}}{1-\varrho_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x} \left(\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}} \right] \end{array} \right) \tag{11}$$

Proof:

$$\bigotimes_{\ell=1}^{x} \alpha'_{\mathfrak{r}_\ell} = \left(\begin{array}{c} \left[\frac{1}{1+\left(\sum_{\ell=1}^{x} \left(\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}} \right)^{\tau} \right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\sum_{\ell=1}^{x} \left(\frac{1-\beta_{\mathfrak{r}_\ell}^{n}}{\beta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau} \right)^{\frac{1}{\tau}}} \right] \\ \left[1 - \frac{1}{1+\left(\sum_{\ell=1}^{x} \left(\frac{\gamma_{\mathfrak{r}_\ell}^{n}}{1-\gamma_{\mathfrak{r}_\ell}^{n}} \right)^{\tau} \right)^{\frac{1}{\tau}}}, 1 - \frac{1}{1+\left(\sum_{\ell=1}^{x} \left(\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau} \right)^{\frac{1}{\tau}}} \right] \\ \left[1 - \frac{1}{1+\left(\sum_{\ell=1}^{x} \left(\frac{\varrho_{\mathfrak{r}_\ell}^{n}}{1-\varrho_{\mathfrak{r}_\ell}^{n}} \right)^{\tau} \right)^{\frac{1}{\tau}}}, 1 - \frac{1}{1+\left(\sum_{\ell=1}^{x} \left(\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau} \right)^{\frac{1}{\tau}}} \right] \end{array} \right) \tag{12}$$

$$\left(\bigotimes_{\ell=1}^{x} \alpha'_{\mathfrak{r}_\ell}\right)^{\frac{1}{x}} = \left(\begin{array}{c} \left[\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{1-\beta_{\mathfrak{r}_\ell}^{n}}{\beta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{\gamma_{\mathfrak{r}_\ell}^{n}}{1-\gamma_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{\varrho_{\mathfrak{r}_\ell}^{n}}{1-\varrho_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \end{array}\right) \quad (13)$$

$$\bigoplus_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}} \left(\bigotimes_{\ell=1}^{x} \alpha'_{\mathfrak{r}_\ell}\right)^{\frac{1}{x}} = \left(\begin{array}{c} \left[1-\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\beta_{\mathfrak{r}_\ell}^{n}}{\beta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{\gamma_{\mathfrak{r}_\ell}^{n}}{1-\gamma_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{\varrho_{\mathfrak{r}_\ell}^{n}}{1-\varrho_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right] \end{array}\right) \quad (14)$$

$$IVT-SPFDHM^{(x)}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathfrak{h}}) = \frac{\bigoplus_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\left(\bigotimes_{\ell=1}^{x} {\alpha'_{\mathfrak{r}_\ell}}^{n}\right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^{x}} \quad (15)$$

$$= \left(\begin{array}{c} \left[1 - \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x}\left(\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, 1 - \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x}\left(\frac{1-\beta_{\mathfrak{r}_\ell}^{n}}{\beta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}} \right] \\ \left[\frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x}\left(\frac{\gamma_{\mathfrak{r}_\ell}^{n}}{1-\gamma_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x}\left(\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}} \right] \\ \left[\frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x}\left(\frac{\varrho_{\mathfrak{r}_\ell}^{n}}{1-\varrho_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\frac{x}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{1}{\sum_{\ell=1}^{x}\left(\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}} \right] \end{array} \right) \tag{16}$$

Example 1: Let $\alpha_1' = ([0.6,0.92],[0.2,0.3],[0.3,0.5])$ $\alpha_2' = ([0.77,0.8],[0.11,0.23][0.10,0.11])$, $\alpha_3' = ([0.66,0.73],[0.18,0.25][0.2,0.4])$, and $\alpha_4' = ([0.56,0.82],[0.11,0.23],[0.16,0.22])$ which is four IVT-SFNs and $x = 2, \tau = 3, n = 3$.

$$\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}} = (3.6296, 1.1904, 2.4783, 4.6942)$$

$$\frac{1-\gamma_{\mathfrak{r}_\ell}^{n}}{\gamma_{\mathfrak{r}_\ell}^{n}} = (124.0000, 750.3148, 170.4678, 750.3148)$$

$$\frac{1-\varrho_{\mathfrak{r}_\ell}^{n}}{\varrho_{\mathfrak{r}_\ell}^{n}} = (36.0370, 999.000, 124.0000, 243.1406)$$

$$\frac{\beta_{\mathfrak{r}_\ell}^{n}}{1-\beta_{\mathfrak{r}_\ell}^{n}} = (3.5185, 1.0492, 0.6367, 1.2290)$$

$$\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}} = (0.0277, 0.0123, 0.015873, 0.012317)$$

$$\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}} = (0.1429, 0.0013, 0.0684, 0.0108)$$

$$IVT-SPFDHM^{(x)}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathfrak{h}})=\frac{\oplus_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\overset{x}{\underset{\ell=1}{\otimes}}\alpha'_{\mathfrak{r}_\ell}{}^{n}\right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^{x}}$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}1-\frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(3.6296)^3+(1.1904)^3}+\frac{1}{(3.6296)^3+(2.4783)^3}+\frac{1}{(3.6296)^3+(4.6942)^3}\\+\frac{1}{(1.1904)^3+(2.4783)^3}+\frac{1}{(1.1904)^3+(4.6942)^3}+\frac{1}{(2.4783)^3+(4.6942)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)},\\ 1/\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(124.0000)^3+(750.3148)^3}+\frac{1}{(124.0000)^3+(170.4678)^3}+\frac{1}{(124.0000)^3+(750.3148)^3}\\+\frac{1}{(750.3148)^3+(170.4678)^3}+\frac{1}{(750.3148)^3+(750.3148)^3}+\frac{1}{(170.4678)^3+(750.3148)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)\end{array}\right]\\ \left[\begin{array}{c}1-\frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(36.0370)^3+(999.000)^3}+\frac{1}{(36.0370)^3+(124.0000)^3}+\frac{1}{(36.0370)^3+(243.1406)^3}\\+\frac{1}{(999.000)^3+(124.0000)^3}+\frac{1}{(999.000)^3+(243.1406)^3}+\frac{1}{(124.0000)^3+(243.1406)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)},\\ \frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(3.5185)^3+(1.0492)^3}+\frac{1}{(3.5185)^3+(0.6367)^3}+\frac{1}{(3.5185)^3+(1.2290)^3}\\+\frac{1}{(1.0492)^3+(0.6367)^3}+\frac{1}{(1.0492)^3+(1.2290)^3}+\frac{1}{(0.6367)^3+(1.2290)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)}\end{array}\right]\\ \left[\begin{array}{c}1-\frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(0.0277)^3+(0.0123)^3}+\frac{1}{(0.0277)^3+(0.015873)^3}+\frac{1}{(0.0277)^3+(0.012317)^3}\\+\frac{1}{(0.0123)^3+(0.015873)^3}+\frac{1}{(0.0123)^3+(0.012317)^3}+\frac{1}{(0.015873)^3+(0.012317)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)},\\ 1/\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(0.1429)^3+(0.0013)^3}+\frac{1}{(0.1429)^3+(0.0684)^3}+\frac{1}{(0.1429)^3+(0.0108)^3}\\+\frac{1}{(0.0013)^3+(0.0684)^3}+\frac{1}{(0.0013)^3+(0.0108)^3}+\frac{1}{(0.0684)^3+(0.0108)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)\end{array}\right]\end{array}\right)$$

$$=[0.2547,0.4468][0.0157,0.9963][0.0152,0.9940]$$

Property 1: $(Idempotecy)$ If $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\dots,\mathfrak{h})=\alpha$ are equal, then $IVT-SPFWDHM^{(x)}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathfrak{h}})=\alpha$

Property 2: $(Monotonicity)$ If $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\dots,\mathfrak{h})$ and $\theta'_\ell=([r_\ell{}^n,p_\ell{}^n],[m_\ell{}^n,l_\ell{}^n],[s_\ell{}^n,d_\ell{}^n])(\ell=1,2,\dots,\mathfrak{h})$ are two sets of IVT-SPFNs. If $\alpha_\ell{}^n\le r_\ell{}^n,\gamma_\ell{}^n\le m_\ell{}^n,\ \varrho_\ell{}^n\le s_\ell{}^n$ and $\beta_\ell{}^n\ge p_\ell{}^n,\ \delta_\ell{}^n\ge l_\ell{}^n,\ \vartheta_\ell{}^n\ge d_\ell{}^n$ apply to all ℓ, then

$IVT-SPFDHM^{(x)}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathfrak{h}})\le IVT-SPFDHM^{(x)}(\theta'_1,\theta'_2,\dots,\theta'_{\mathfrak{h}})$

Property 3: $(Boundedness)$ Let $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\dots,\mathfrak{h})$ be a set of IVT-SPFNs. If $\alpha'_{\mathfrak{r}}{}^{+}=\begin{pmatrix}[max_{\mathfrak{r}}(\alpha_\ell{}^n),max_{\mathfrak{r}}(\gamma_\ell{}^n),max_{\mathfrak{r}}(\varrho_\ell{}^n)],\\ [min_{\mathfrak{r}}(\beta_\ell{}^n),min_{\mathfrak{r}}(\delta_\ell{}^n),min_{\mathfrak{r}}(\vartheta_\ell{}^n)]\end{pmatrix}$ and If $\alpha'_{\mathfrak{r}}{}^{-}=\begin{pmatrix}[min_{\mathfrak{r}}(\alpha_\ell{}^n),min_{\mathfrak{r}}(\gamma_\ell{}^n),min_{\mathfrak{r}}(\varrho_\ell{}^n)],\\ [max_{\mathfrak{r}}(\beta_\ell{}^n),max_{\mathfrak{r}}(\delta_\ell{}^n),max_{\mathfrak{r}}(\vartheta_\ell{}^n)]\end{pmatrix}$

Then

$$\alpha'^{-}\le IVT-SPFDHM^{(x)}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathfrak{h}})\le\alpha'^{+}$$

3.2 The IVT-SPFWDHM Operator

It is crucial to pay attention to attribute weights in a real MADM. The interval-valued IVT-SPFWDHM operator is what we suggest as a result.

Definition 10. Let $\alpha'_\ell = ([\alpha_\ell{}^n, \beta_\ell{}^n], [\gamma_\ell{}^n, \delta_\ell{}^n], [\varrho_\ell{}^n, \vartheta_\ell{}^n])(\ell = 1,2,\dots,\mathfrak{h})$ be a set of IVT-SPFNs with their weight vector $w_\mathrm{r} = (w_1, w_2, \dots, w_\mathfrak{h})^T$, thereby satisfying $w_\mathrm{r} \in [0,1]$ and $\sum_{\mathrm{r}=1}^{\mathfrak{h}} w_\mathrm{r} = 1$. Then the IVT-SPFWDHM operator is as follows:

$$IVT-SPFWDHM_w^x(\alpha'_1, \alpha'_2, \dots, \alpha'_\mathfrak{h}) = \frac{\oplus_{1\le \mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\left(\overset{x}{\underset{\ell=1}{\otimes}}\left((\alpha'_{\mathrm{r}\ell})^{w_{\mathrm{r}_\ell}}\right)\right)^{\frac{1}{x}}}{C_\mathfrak{h}^x} \tag{17}$$

Theorem 2: Let $\alpha'_\ell = ([\alpha_\ell{}^n, \beta_\ell{}^n], [\gamma_\ell{}^n, \delta_\ell{}^n], [\varrho_\ell{}^n, \vartheta_\ell{}^n])(\ell = 1,2,\dots,\mathfrak{h})(\ell = 1,2,\dots,\mathfrak{h})$ be a set of IVT-SPFNs. The IVT-SPFWDHM operator is also an IVT-SPFN where

$$IVT-SPFDHM_w^x(\alpha'_1, \alpha'_2, \dots, \alpha'_\mathfrak{h}) = \frac{\oplus_{1\le \mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\left(\overset{x}{\underset{\ell=1}{\otimes}}\left((\alpha'_{\mathrm{r}\ell})^{w_{\mathrm{r}_\ell}}\right)\right)^{\frac{1}{x}}}{C_\mathfrak{h}^x} \tag{18}$$

$$= \left(\begin{array}{c} \left[1-\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{1-\alpha^n_{\mathrm{r}_\ell}}{\alpha^n_{\mathrm{r}_\ell}}\right)^\tau}\right]^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{1-\beta^n_{\mathrm{r}_\ell}}{\beta^n_{\mathrm{r}_\ell}}\right)^\tau}\right]^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\gamma^n_{\mathrm{r}_\ell}}{1-\gamma^n_{\mathrm{r}_\ell}}\right)^\tau}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\delta^n_{\mathrm{r}_\ell}}{1-\delta^n_{\mathrm{r}_\ell}}\right)^\tau}\right]^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\varrho^n_{\mathrm{r}_\ell}}{1-\varrho^n_{\mathrm{r}_\ell}}\right)^\tau}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\vartheta^n_{\mathrm{r}_\ell}}{1-\vartheta^n_{\mathrm{r}_\ell}}\right)^\tau}\right]^{\frac{1}{\tau}}}\right] \end{array}\right) \tag{19}$$

Proof:

$$\left(\alpha'_{\mathrm{r}_\ell}\right)^{w_{\mathrm{r}_\ell}} = \begin{pmatrix} \left[\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{1-\alpha^n_{\mathrm{r}_\ell}}{\alpha^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{1-\beta^n_{\mathrm{r}_\ell}}{\beta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{\gamma^n_{\mathrm{r}_\ell}}{1-\gamma^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{\delta^n_{\mathrm{r}_\ell}}{1-\delta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{\varrho^n_{\mathrm{r}_\ell}}{1-\varrho^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{\vartheta^n_{\mathrm{r}_\ell}}{1-\vartheta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \end{pmatrix} \tag{20}$$

$$\bigotimes_{\ell=1}^{x}\left(\alpha'_{\mathrm{r}_\ell}\right)^{w_{\mathrm{r}_\ell}} = \begin{pmatrix} \left[\frac{1}{1+\left(\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{1-\alpha^n_{\mathrm{r}_\ell}}{\alpha^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{1-\beta^n_{\mathrm{r}_\ell}}{\beta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\gamma^n_{\mathrm{r}_\ell}}{1-\gamma^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\delta^n_{\mathrm{r}_\ell}}{1-\delta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\varrho^n_{\mathrm{r}_\ell}}{1-\varrho^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\vartheta^n_{\mathrm{r}_\ell}}{1-\vartheta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \end{pmatrix} \tag{21}$$

$$\left(\bigotimes_{\ell=1}^{x}\left(\alpha'_{\mathrm{r}_\ell}\right)^{w_{\mathrm{r}_\ell}}\right)^{\frac{1}{x}} = \begin{pmatrix} \left[\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{1-\alpha^n_{\mathrm{r}_\ell}}{\alpha^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{1-\beta^n_{\mathrm{r}_\ell}}{\beta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\gamma^n_{\mathrm{r}_\ell}}{1-\gamma^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\delta^n_{\mathrm{r}_\ell}}{1-\delta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\varrho^n_{\mathrm{r}_\ell}}{1-\varrho^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^x w_{\mathrm{r}_\ell}\left(\frac{\vartheta^n_{\mathrm{r}_\ell}}{1-\vartheta^n_{\mathrm{r}_\ell}}\right)^\tau\right)^{\frac{1}{\tau}}}\right] \end{pmatrix} \tag{22}$$

$$\oplus_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\left(\bigotimes_{\ell=1}^{x}\left({\alpha'}_{\mathrm{r}_\ell}\right)^{w_{\mathrm{r}_\ell}}\right)^{\frac{1}{x}}=$$

$$\left(\begin{array}{c}\left[1-\frac{1}{1+\left[\sum_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\alpha_{\mathrm{r}_\ell}^{n}}{\alpha_{\mathrm{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\sum_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\beta_{\mathrm{r}_\ell}^{n}}{\beta_{\mathrm{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left[\sum_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\gamma_{\mathrm{r}_\ell}^{n}}{1-\gamma_{\mathrm{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},\frac{1}{1+\left[\sum_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\delta_{\mathrm{r}_\ell}^{n}}{1-\delta_{\mathrm{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left[\sum_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\varrho_{\mathrm{r}_\ell}^{n}}{1-\varrho_{\mathrm{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},\frac{1}{1+\left[\sum_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\vartheta_{\mathrm{r}_\ell}^{n}}{1-\vartheta_{\mathrm{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\end{array}\right) \quad (23)$$

$$IVT-SPFDHM^{(x)}\left(\alpha'_1,\alpha'_2,\ldots,\alpha'_{\mathfrak{h}}\right)=\frac{\oplus_{1\leq \mathrm{r}_1<\cdots<\mathrm{r}_x\leq \mathfrak{h}}\left(\bigotimes_{\ell=1}^{x}\left({\alpha'}_{\mathrm{r}_\ell}\right)^{w_{\mathrm{r}_\ell}}\right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^{x}} \quad (24)$$

$$= \left(\begin{array}{c} \left[1 - \frac{1}{1 + \left[\frac{1}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x} w_{\mathfrak{r}_\ell} \left(\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}}, 1 - \frac{1}{1 + \left[\frac{1}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x} w_{\mathfrak{r}_\ell} \left(\frac{1-\beta_{\mathfrak{r}_\ell}^{n}}{\beta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}} \right] \\ \left[\frac{1}{1 + \left[\frac{1}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x} w_{\mathfrak{r}_\ell} \left(\frac{\gamma_{\mathfrak{r}_\ell}^{n}}{1-\gamma_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}}, \frac{1}{1 + \left[\frac{1}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x} w_{\mathfrak{r}_\ell} \left(\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}} \right] \\ \left[\frac{1}{1 + \left[\frac{1}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x} w_{\mathfrak{r}_\ell} \left(\frac{\varrho_{\mathfrak{r}_\ell}^{n}}{1-\varrho_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}}, \frac{1}{1 + \left[\frac{1}{C_{\mathfrak{h}}^{x}} \sum_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x} w_{\mathfrak{r}_\ell} \left(\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}} \right)^{\tau}} \right]^{\frac{1}{\tau}}} \right] \end{array} \right) \quad (25)$$

Example 2: Let $\alpha_1' = ([0.6,0.92],[0.2,0.3],[0.3,0.5])$ $\alpha_2' = ([0.77,0.8],[0.11,0.23][0.10,0.11])$, $\alpha_3' = ([0.66,0.73],[0.18,0.25][0.2,0.4])$, and $\alpha_4' = ([0.56,0.82],[0.11,0.23],[0.16,0.22])$ be four IVT-SPFNs and $x = 2, \tau = 3$, $n = 3$ and $Weight =$ 0.3467,0.2521,0.2207,0.1805

$$\frac{1-\alpha_{\mathfrak{r}_\ell}^{n}}{\alpha_{\mathfrak{r}_\ell}^{n}} = (3.6296, 1.1904, 2.4783, 4.6942)$$

$$\frac{1-\gamma_{\mathfrak{r}_\ell}^{n}}{\gamma_{\mathfrak{r}_\ell}^{n}} = (124.0000, 750.3148, 170.4678, 750.3148)$$

$$\frac{1-\varrho_{\mathfrak{r}_\ell}^{n}}{\varrho_{\mathfrak{r}_\ell}^{n}} = (36.0370, 999.000, 124.0000, 243.1406)$$

$$\frac{\beta_{\mathfrak{r}_\ell}^{n}}{1-\beta_{\mathfrak{r}_\ell}^{n}} = (3.5185, 1.0492, 0.6367, 1.2290)$$

$$\frac{\delta_{\mathfrak{r}_\ell}^{n}}{1-\delta_{\mathfrak{r}_\ell}^{n}} = (0.0277, 0.0123, 0.015873, 0.012317)$$

$$\frac{\vartheta_{\mathfrak{r}_\ell}^{n}}{1-\vartheta_{\mathfrak{r}_\ell}^{n}} = (0.1429, 0.0013, 0.0684, 0.0108)$$

$$IVT-SPFWDHM^{(x)}\left(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathfrak{h}}\right)=\frac{\oplus_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\overset{x}{\underset{\ell=1}{\otimes}}{\alpha'_{\mathfrak{r}_\ell}}^{n}\right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^{x}}$$

$$=\left(\begin{matrix}\left[\begin{matrix}1-\dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{matrix}\frac{1}{0.2740\times(3.6296)^3+0.2564\times(1.1904)^3}+\frac{1}{0.2740\times(3.6296)^3+0.2524\times(2.4783)^3}+\frac{1}{0.2740\times(3.6296)^3+0.2173\times(4.6942)^3}\\+\frac{1}{0.2564\times(1.1904)^3+0.2524\times(2.4783)^3}+\frac{1}{0.2564\times(1.1904)^3+0.2173\times(4.6942)^3}+\frac{1}{0.2524\times(2.4783)^3+0.2173\times(4.6942)^3}\end{matrix}\right)\right)^{\frac{1}{3}}\right)},\\ 1/\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{matrix}\frac{1}{0.2740\times(124.0000)^3+0.2564\times(750.3148)^3}+\frac{1}{0.2740\times(124.0000)^3+0.2524\times(170.4678)^3}+\frac{1}{0.2740\times(124.0000)^3+0.2173\times(750.3148)^3}\\+\frac{1}{0.2564\times(750.3148)^3+0.2524\times(170.4678)^3}+\frac{1}{0.2564\times(750.3148)^3+0.2173\times(750.3148)^3}+\frac{1}{0.2524\times(170.4678)^3+0.2173\times(750.3148)^3}\end{matrix}\right)\right)^{\frac{1}{3}}\right)\end{matrix}\right]\\ \left[\begin{matrix}1-\dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{matrix}\frac{1}{0.2740\times(36.0370)^3+0.2564\times(999.000)^3}+\frac{1}{0.2740\times(36.0370)^3+0.2524\times(124.0000)^3}+\frac{1}{0.2740\times(36.0370)^3+0.2173\times(243.1406)^3}\\+\frac{1}{0.2564\times(999.000)^3+0.2524\times(124.0000)^3}+\frac{1}{0.2564\times(999.000)^3+0.2173\times(243.1406)^3}+\frac{1}{0.2524\times(124.0000)^3+0.2173\times(243.1406)^3}\end{matrix}\right)\right)^{\frac{1}{3}}\right)},\\ \dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{matrix}\frac{1}{0.2740\times(3.5185)^3+0.2564\times(1.0492)^3}+\frac{1}{0.2740\times(3.5185)^3+0.2524\times(0.6367)^3}+\frac{1}{0.2740\times(3.5185)^3+0.2173\times(1.2290)^3}\\+\frac{1}{0.2564\times(1.0492)^3+0.2524\times(0.6367)^3}+\frac{1}{0.2564\times(1.0492)^3+0.2173\times(1.2290)^3}+\frac{1}{0.2524\times(0.6367)^3+0.2173\times(1.2290)^3}\end{matrix}\right)\right)^{\frac{1}{3}}\right)}\end{matrix}\right]\\ \left[\begin{matrix}1-\dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{matrix}\frac{1}{0.2740\times(0.0277)^3+0.2564\times(0.0123)^3}+\frac{1}{0.2740\times(0.0277)^3+0.2524\times(0.015873)^3}+\frac{1}{0.2740\times(0.0277)^3+0.2173\times(0.012317)^3}\\+\frac{1}{0.2564\times(0.0123)^3+0.2524\times(0.015873)^3}+\frac{1}{0.2564\times(0.0123)^3+0.2173\times(0.012317)^3}+\frac{1}{0.2524\times(0.015873)^3+0.2173\times(0.012317)^3}\end{matrix}\right)\right)^{\frac{1}{3}}\right)},\\ 1/\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{matrix}\frac{1}{0.2740\times(0.1429)^3+0.2564\times(0.0013)^3}+\frac{1}{0.2740\times(0.1429)^3+0.2524\times(0.0684)^3}+\frac{1}{0.2740\times(0.1429)^3+0.2173\times(0.0108)^3}\\+\frac{1}{0.2564\times(0.0013)^3+0.2524\times(0.0684)^3}+\frac{1}{0.2564\times(0.0013)^3+0.2173\times(0.0108)^3}+\frac{1}{0.2524\times(0.0684)^3+0.2173\times(0.0108)^3}\end{matrix}\right)\right)^{\frac{1}{3}}\right)\end{matrix}\right]\end{matrix}\right)$$

$$=[0.3513,0.5651][0.0099,0.9942][0.00992,0.9905]$$

Property 4: $(Monotonicity)$ If $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\dots,\mathfrak{h})$ and $\theta'_\ell=([r_\ell{}^n,p_\ell{}^n],[m_\ell{}^n,l_\ell{}^n],[s_\ell{}^n,d_\ell{}^n])(\ell=1,2,\dots,\mathfrak{h})$ be two sets of IVT-SFNs. If $\alpha_\ell{}^n\le r_\ell{}^n$, $\gamma_\ell{}^n\le m_\ell{}^n$, $\varrho_\ell{}^n\le s_\ell{}^n$and $\beta_\ell{}^n\ge p_\ell{}^n$, $\delta_\ell{}^n\ge l_\ell{}^n$, $\vartheta_\ell{}^n\ge d_\ell{}^n$ apply to all ℓ, then

$$IVT-SPFWDHM^{(x)}\left(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathfrak{h}}\right)\le IVT-SPFWDHM^{(x)}\left(\theta'_1,\theta'_2,\dots,\theta'_{\mathfrak{h}}\right)$$

Property 5: $(Boundedness)$ Let $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\dots,\mathfrak{h})$ be a set of IVT-SFNs. If ${\alpha'_{\mathfrak{r}}}^{+}=\begin{pmatrix}[max_{\mathfrak{r}}(\alpha_\ell{}^n),max_{\mathfrak{r}}(\gamma_\ell{}^n),max_{\mathfrak{r}}(\varrho_\ell{}^n)],\\ [min_{\mathfrak{r}}(\beta_\ell{}^n),min_{\mathfrak{r}}(\delta_\ell{}^n),min_{\mathfrak{r}}(\vartheta_\ell{}^n)]\end{pmatrix}$ and If ${\alpha'_{\mathfrak{r}}}^{-}=\begin{pmatrix}[min_{\mathfrak{r}}(\alpha_\ell{}^n),min_{\mathfrak{r}}(\gamma_\ell{}^n),min_{\mathfrak{r}}(\varrho_\ell{}^n)],\\ [max_{\mathfrak{r}}(\beta_\ell{}^n),max_{\mathfrak{r}}(\delta_\ell{}^n),max_{\mathfrak{r}}(\vartheta_\ell{}^n)]\end{pmatrix}$

Then

$$\alpha'^{-}\le IVT-SPFWDHM^{(x)}\left(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathfrak{h}}\right)\le\alpha'^{+}$$

3.3 The IVT-SPFDDHM Operator:

The IVT-SPFWDHM Operator (Wu et al., 2018) proposed the dual HM(DHM) operator.

Definition 11. (Wu et al., 2018): The DHM operator is as follows:

$$DHM^{(x)}\left(\varphi_1,\varphi_2,\dots,\varphi_{\mathfrak{h}}\right)=\left(\prod_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\frac{\sum_{\ell=1}^{x}\varphi_{\mathfrak{r}_\ell}}{x}\right)\right)^{\frac{1}{C_{\mathfrak{h}}^{x}}}\tag{26}$$

where x is a parameter and $x=1,2,\dots,\mathfrak{h},\mathfrak{r}_1,\mathfrak{r}_2,\dots,\mathfrak{r}_x$ are x integer values taken from the set $\{1,2,\dots,\mathfrak{h}\}$ of k integer values, $C_{\mathfrak{h}}^{x}$ denotes the binomial coefficient and $C_{\mathfrak{h}}^{x}=\frac{\mathfrak{h}!}{x!(\mathfrak{h}-x)!}$.

In this section, we will propose the interval values intuitionistic fuzzy Dombi DHM $(IVT-SPFDHM)$ operator.

Definition 12. Let $\alpha'_\ell = ([\alpha_\ell{}^n, \beta_\ell{}^n], [\gamma_\ell{}^n, \delta_\ell{}^n], [\varrho_\ell{}^n, \vartheta_\ell{}^n])(\ell = 1,2,\ldots,\mathfrak{h})$ be a set of IVSFNs. The IVT-SPFDDHM operator is

$$IVT-SPFDDHM^{(x)}(\alpha'_1, \alpha'_2, \ldots, \alpha'_\mathfrak{h}) = \left(\bigotimes_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\frac{\bigoplus_{\ell=1}^{x}\alpha'_{\mathfrak{r}_\ell}}{x}\right)\right)^{\frac{1}{C_\mathfrak{h}^x}} \tag{27}$$

Theorem 3: Let $\alpha'_\ell = ([\alpha_\ell, \beta_\ell], [\gamma_\ell, \delta_\ell], [\varrho_\ell, \vartheta_\ell])(\ell = 1,2,\ldots,\mathfrak{h})$ be a set of IVT-SPFNs. The IVT-SPFDDHM operator is also an IVT-SPFN where

$$IVT-SPFDDHM^{(x)}(\alpha'_1, \alpha'_2, \ldots, \alpha'_\mathfrak{h}) = \left(\bigotimes_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\frac{\bigoplus_{\ell=1}^{x}\alpha'_{\mathfrak{r}_\ell}}{x}\right)\right)^{\frac{1}{C_\mathfrak{h}^x}} \tag{28}$$

$$= \left(\begin{array}{c}\left[\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{\alpha_{\mathfrak{r}_\ell}^n}{1-\alpha_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{\beta_{\mathfrak{r}_\ell}^n}{1-\beta_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\gamma_{\mathfrak{r}_\ell}^n}{\gamma_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\delta_{\mathfrak{r}_\ell}^n}{\delta_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}\right] \\ \left[1-\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\varrho_{\mathfrak{r}_\ell}^n}{\varrho_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left[\frac{1}{C_\mathfrak{h}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\vartheta_{\mathfrak{r}_\ell}^n}{\vartheta_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}\right]\end{array}\right) \tag{29}$$

Proof:

$$\overset{x}{\underset{\ell=1}{\oplus}} \alpha'_{r_\ell} = \left(\begin{array}{c} \left[1-\frac{1}{1+\left(\sum_{\ell=1}^{x}\left(\frac{\alpha^{n}_{r_\ell}}{1-\alpha^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\sum_{\ell=1}^{x}\left(\frac{\beta^{n}_{r_\ell}}{1-\beta^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\sum_{\ell=1}^{x}\left(\frac{1-\gamma^{n}_{r_\ell}}{\gamma^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\sum_{\ell=1}^{x}\left(\frac{1-\delta^{n}_{r_\ell}}{\delta^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\sum_{\ell=1}^{x}\left(\frac{1-\varrho^{n}_{r_\ell}}{\varrho^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\sum_{\ell=1}^{x}\left(\frac{1-\vartheta^{n}_{r_\ell}}{\vartheta^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \end{array} \right) \tag{30}$$

$$\frac{\overset{x}{\underset{\ell=1}{\oplus}} \alpha'_{r_\ell}}{x} = \left(\begin{array}{c} \left[1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{\alpha^{n}_{r_\ell}}{1-\alpha^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{\beta^{n}_{r_\ell}}{1-\beta^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{1-\gamma^{n}_{r_\ell}}{\gamma^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{1-\delta^{n}_{r_\ell}}{\delta^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{1-\varrho^{n}_{r_\ell}}{\varrho^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}, \frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}\left(\frac{1-\vartheta^{n}_{r_\ell}}{\vartheta^{n}_{r_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right] \end{array} \right) \tag{31}$$

$$\otimes_{1\le r_1<\cdots<r_x\le \mathfrak{h}} \left(\frac{\overset{x}{\underset{\ell=1}{\oplus}} \alpha'_{r_\ell}}{x} \right) = \left(\begin{array}{c} \left[\frac{1}{1+\left[\sum_{1\le r_1<\cdots<r_x\le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x}\left(\frac{\alpha^{n}_{r_\ell}}{1-\alpha^{n}_{r_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, 1-\frac{1}{1+\left[\sum_{1\le r_1<\cdots<r_x\le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x}\left(\frac{\beta^{n}_{r_\ell}}{1-\beta^{n}_{r_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left[\sum_{1\le r_1<\cdots<r_x\le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\gamma^{n}_{r_\ell}}{\gamma^{n}_{r_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\sum_{1\le r_1<\cdots<r_x\le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\delta^{n}_{r_\ell}}{\delta^{n}_{r_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right] \\ \left[\frac{1}{1+\left[\sum_{1\le r_1<\cdots<r_x\le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\varrho^{n}_{r_\ell}}{\varrho^{n}_{r_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}, \frac{1}{1+\left[\sum_{1\le r_1<\cdots<r_x\le \mathfrak{h}} \frac{x}{\sum_{\ell=1}^{x}\left(\frac{1-\vartheta^{n}_{r_\ell}}{\vartheta^{n}_{r_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right] \end{array} \right) \tag{32}$$

$$IVT-SPFDDHM^{(x)}\left(\varphi'_1,\varphi'_2,\dots,\varphi'_{\mathfrak{h}}\right)=\left(\bigotimes_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\left(\frac{\overset{x}{\underset{\ell=1}{\oplus}}\varphi'_{\mathfrak{r}_\ell}}{x}\right)\right)^{\frac{1}{C_{\mathfrak{h}}^x}}$$

$$=\left(\begin{array}{c}\left[\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x\left(\frac{\alpha_{\mathfrak{r}_\ell}^n}{1-\alpha_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}},\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x\left(\frac{\beta_{\mathfrak{r}_\ell}^n}{1-\beta_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x\left(\frac{1-\gamma_{\mathfrak{r}_\ell}^n}{\gamma_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x\left(\frac{1-\delta_{\mathfrak{r}_\ell}^n}{\delta_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x\left(\frac{1-\varrho_{\mathfrak{r}_\ell}^n}{\varrho_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathfrak{r}_1<\cdots<\mathfrak{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^x\left(\frac{1-\vartheta_{\mathfrak{r}_\ell}^n}{\vartheta_{\mathfrak{r}_\ell}^n}\right)^\tau}\right]^{\frac{1}{\tau}}}\right]\end{array}\right) \quad (33)$$

Example 3: Let $\alpha'_1=([0.6,0.92],[0.2,0.3],[0.3,0.5])$ $\alpha'_2=([0.77,0.8],[0.11,0.23][0.10,0.11])$, $\alpha'_3=([0.66,0.73],[0.18,0.25][0.2,0.4])$, and $\alpha'_4=([0.56,0.82],[0.11,0.23],[0.16,0.22])$ be four IVT-SPFNs and $x=2,\tau=3,n=3$.

$$\frac{\alpha_{\mathfrak{r}_\ell}^n}{1-\alpha_{\mathfrak{r}_\ell}^n}=(0.2842,0.9531,1.5706,0.8137)$$

$$\frac{\gamma_{\mathfrak{r}_\ell}^n}{1-\gamma_{\mathfrak{r}_\ell}^n}=(36.0370,81.1895,63.0000,81.1895)$$

$$\frac{\varrho_{\mathfrak{r}_\ell}^n}{1-\varrho_{\mathfrak{r}_\ell}^n}=(7.0000,750.3148,14.6250,92.9144)$$

$$\frac{1-\beta_{\mathfrak{r}_\ell}^n}{\beta_{\mathfrak{r}_\ell}^n}=(0.2755,0.8400,0.4035,0.2130)$$

$$\frac{1-\delta_{\mathfrak{r}_\ell}^n}{\delta_{\mathfrak{r}_\ell}^n}=(0.0081,0.0013,0.005866,0.001333)$$

$$\frac{1-\vartheta_{\mathfrak{r}_\ell}^n}{\vartheta_{\mathfrak{r}_\ell}^n}=(0.0277,0.0010,0.0081,0.0041)$$

$$IVT-SPFDHM^{(x)}\left(\alpha'_1,\alpha'_2,\ldots,\alpha'_{\mathfrak{h}}\right)=\frac{\oplus_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\left(\underset{\ell=1}{\overset{x}{\otimes}}\alpha'_{\mathfrak{r}_\ell}\right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^{x}}$$

$$=\left(\begin{array}{c}
\left[\begin{array}{c}
\dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(0.2842)^3+(0.9531)^3}+\frac{1}{(0.2842)^3+(1.5706)^3}+\frac{1}{(0.2842)^3+(0.8137)^3}\\+\frac{1}{(0.9531)^3+(1.5706)^3}+\frac{1}{(0.9531)^3+(0.8137)^3}+\frac{1}{(1.5706)^3+(0.8137)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)},\\
1-1/\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(36.0370)^3+(81.1895)^3}+\frac{1}{(36.0370)^3+(63.0000)^3}+\frac{1}{(36.0370)^3+(81.1895)^3}\\+\frac{1}{(81.1895)^3+(63.0000)^3}+\frac{1}{(81.1895)^3+(81.1895)^3}+\frac{1}{(63.0000)^3+(81.1895)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)
\end{array}\right]\\
\left[\begin{array}{c}
\dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(7.0000)^3+(750.3148)^3}+\frac{1}{(7.0000)^3+(14.6250)^3}+\frac{1}{(7.0000)^3+(92.9144)^3}\\+\frac{1}{(750.3148)^3+(14.6250)^3}+\frac{1}{(750.3148)^3+(92.9144)^3}+\frac{1}{(14.6250)^3+(92.9144)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)},\\
1-\dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(0.2755)^3+(0.8400)^3}+\frac{1}{(0.2755)^3+(0.4035)^3}+\frac{1}{(0.2755)^3+(0.2130)^3}\\+\frac{1}{(0.8400)^3+(0.4035)^3}+\frac{1}{(0.8400)^3+(0.2130)^3}+\frac{1}{(0.4035)^3+(0.2130)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)}
\end{array}\right]\\
\left[\begin{array}{c}
\dfrac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(0.0081)^3+(0.0013)^3}+\frac{1}{(0.0081)^3+(0.005866)^3}+\frac{1}{(0.0081)^3+(0.001333)^3}\\+\frac{1}{(0.0013)^3+(0.005866)^3}+\frac{1}{(0.0013)^3+(0.001333)^3}+\frac{1}{(0.005866)^3+(0.00133)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)},\\
1-1/\left(1+\left(\frac{2}{C_4^2}\times\left(\begin{array}{c}\frac{1}{(0.0277)^3+(0.0010)^3}+\frac{1}{(0.0277)^3+(0.0081)^3}+\frac{1}{(0.0277)^3+(0.0041)^3}\\+\frac{1}{(0.0010)^3+(0.0081)^3}+\frac{1}{(0.0010)^3+(0.0041)^3}+\frac{1}{(0.0081)^3+(0.0041)^3}\end{array}\right)\right)^{\frac{1}{3}}\right)
\end{array}\right]
\end{array}\right)$$

$$=[0.5302,0.7333][0.0024,0.9851][0.0055,0.9561]$$

Property 6: $(Idempotecy)$ If $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\ldots,\mathfrak{h})=\alpha$ are equal, then $IVT-SPFDDHM^{(x)}(\alpha'_1,\alpha'_2,\ldots,\alpha'_{\mathfrak{h}})=\alpha$

Property 7: $(Monotonicity)$ If $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\ldots,\mathfrak{h})$ and $\theta'_\ell=([r_\ell{}^n,p_\ell{}^n],[m_\ell{}^n,l_\ell{}^n],[s_\ell{}^n,d_\ell{}^n])(\ell=1,2,\ldots,\mathfrak{h})$ are two sets of IVT-SPFNs. If $\alpha_\ell{}^n\le r_\ell{}^n,\gamma_\ell{}^n\le m_\ell{}^n,\ \varrho_\ell{}^n\le s_\ell{}^n$ and $\beta_\ell{}^n\ge p_\ell{}^n,\delta_\ell{}^n\ge l_\ell{}^n,\vartheta_\ell{}^n\ge d_\ell{}^n$ apply to all ℓ, then

$IVT-SPFDDHM^{(x)}(\alpha'_1,\alpha'_2,\ldots,\alpha'_{\mathfrak{h}})\le IVT-SPFDDHM^{(x)}(\theta'_1,\theta'_2,\ldots,\theta'_{\mathfrak{h}})$

Property 8: $(Boundedness)$ Let $\alpha'_\ell=([\alpha_\ell{}^n,\beta_\ell{}^n],[\gamma_\ell{}^n,\delta_\ell{}^n],[\varrho_\ell{}^n,\vartheta_\ell{}^n])(\ell=1,2,\ldots,\mathfrak{h})$ be a set of IVT-SPFNs. If $\alpha'_{\mathfrak{r}}{}^{+}=\begin{pmatrix}[max_{\mathfrak{r}}(\alpha_\ell{}^n),max_{\mathfrak{r}}(\gamma_\ell{}^n),max_{\mathfrak{r}}(\varrho_\ell{}^n)],\\ [min_{\mathfrak{r}}(\beta_\ell{}^n),min_{\mathfrak{r}}(\delta_\ell{}^n),min_{\mathfrak{r}}(\vartheta_\ell{}^n)]\end{pmatrix}$ and If $\alpha'_{\mathfrak{r}}{}^{-}=\begin{pmatrix}[min_{\mathfrak{r}}(\alpha_\ell{}^n),min_{\mathfrak{r}}(\gamma_\ell{}^n),min_{\mathfrak{r}}(\varrho_\ell{}^n)],\\ [max_{\mathfrak{r}}(\beta_\ell{}^n),max_{\mathfrak{r}}(\delta_\ell{}^n),max_{\mathfrak{r}}(\vartheta_\ell{}^n)]\end{pmatrix}$
Then

$$\alpha'^{-}\le IVT-SPFDDHM^{(x)}(\alpha'_1,\alpha'_2,\ldots,\alpha'_{\mathfrak{h}})\le\alpha'^{+}$$

3.4 The IVT-SPFWDDHM Operator

In practical MADM, it is important to pay attention to attribute weights. We propose the interval-valued intuitionistic fuzzy weighted Dombi DHM ($IVT-SPFWDDHM$) operator.

Definition 13. Let $\alpha'_{\ell}=([\alpha_{\ell},\beta_{\ell}],[\gamma_{\ell},\delta_{\ell}],[\varrho_{\ell},\vartheta_{\ell}])$ $(\ell=1,2,\ldots,\mathfrak{h})$ be a set of IVT-SPFNs with their weight vector $w_{\mathrm{r}}=\left(w_1,w_2,\ldots,w_{\mathfrak{h}}\right)^T$, thereby satisfying $w_{\mathrm{r}}\in[0,1]$ and $\sum_{\mathrm{r}=1}^{\mathfrak{h}}w_{\mathrm{r}}=1$. Then the IVT-SPFWDDHM operator is as follows:

$$IVT-SPFDDHM_w^x\left(\varphi'_1,\varphi'_2,\ldots,\varphi'_{\mathfrak{h}}\right)=\left(\otimes_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\left(\frac{\overset{x}{\underset{\ell=1}{\oplus}}w_{\mathrm{r}_\ell}\varphi'_{\mathrm{r}_\ell}}{x}\right)\right)^{\frac{1}{C_{\mathfrak{h}}^x}} \tag{34}$$

Theorem 4: Let $\alpha'_{\ell}=([\alpha_{\ell},\beta_{\ell}],[\gamma_{\ell},\delta_{\ell}],[\varrho_{\ell},\vartheta_{\ell}])(\ell=1,2,\ldots,\mathfrak{h})$ be a set of IVT-SFNs. The IVT-SPFWDDHM operator is also an IVT-SPFN where

$$IVT-SPFDDHM_w^x\left(\varphi'_1,\varphi'_2,\ldots,\varphi'_{\mathfrak{h}}\right)=\left(\otimes_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\left(\frac{\overset{x}{\underset{\ell=1}{\oplus}}w_{\mathrm{r}_\ell}\varphi'_{\mathrm{r}_\ell}}{x}\right)\right)^{\frac{1}{C_{\mathfrak{h}}^x}}$$

$$=\left(\begin{array}{c}\left[\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\alpha_{\mathrm{r}_\ell}^n}{1-\alpha_{\mathrm{r}_\ell}^n}\right)^{\tau}}\right]^{\frac{1}{\tau}}},\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\beta_{\mathrm{r}_\ell}^n}{1-\beta_{\mathrm{r}_\ell}^n}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\gamma_{\mathrm{r}_\ell}}{\gamma_{\mathrm{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\delta_{\mathrm{r}_\ell}}{\delta_{\mathrm{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\varrho_{\mathrm{r}_\ell}}{\varrho_{\mathrm{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^x}\sum_{1\le\mathrm{r}_1<\cdots<\mathrm{r}_x\le\mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\vartheta_{\mathrm{r}_\ell}}{\vartheta_{\mathrm{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\end{array}\right) \tag{35}$$

Proof:

$$w_{\mathrm{r}_\ell}{\alpha'}_{\mathrm{r}_\ell}=\left(\begin{array}{c}\left[1-\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{\alpha_{\mathrm{r}_\ell}^{n}}{1-\alpha_{\mathrm{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}},1-\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{\beta_{\mathrm{r}_\ell}^{n}}{1-\beta_{\mathrm{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{1-\gamma_{\mathrm{r}_\ell}}{\gamma_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}},\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{1-\delta_{\mathrm{r}_\ell}}{\delta_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{1-\varrho_{\mathrm{r}_\ell}}{\varrho_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}},\frac{1}{1+\left(w_{\mathrm{r}_\ell}\left(\frac{1-\vartheta_{\mathrm{r}_\ell}}{\vartheta_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\end{array}\right) \tag{36}$$

$$\underset{\ell=1}{\overset{x}{\oplus}} w_{\mathrm{r}_\ell}{\alpha'}_{\mathrm{r}_\ell}=\left[\begin{array}{c}\left[1-\frac{1}{1+\left(\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\alpha_{\mathrm{r}_\ell}^{n}}{1-\alpha_{\mathrm{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}},1-\frac{1}{1+\left(\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\beta_{\mathrm{r}_\ell}^{n}}{1-\beta_{\mathrm{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left(\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\gamma_{\mathrm{r}_\ell}}{\gamma_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}},\frac{1}{1+\left(\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\delta_{\mathrm{r}_\ell}}{\delta_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left(\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\varrho_{\mathrm{r}_\ell}}{\varrho_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}},\frac{1}{1+\left(\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\vartheta_{\mathrm{r}_\ell}}{\vartheta_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\end{array}\right] \tag{37}$$

$$\frac{\underset{\ell=1}{\overset{x}{\oplus}} w_{\mathrm{r}_\ell}{\alpha'}_{\mathrm{r}_\ell}}{x}=\left(\begin{array}{c}\left[1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\alpha_{\mathrm{r}_\ell}^{n}}{1-\alpha_{\mathrm{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}},1-\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{\beta_{\mathrm{r}_\ell}^{n}}{1-\beta_{\mathrm{r}_\ell}^{n}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\gamma_{\mathrm{r}_\ell}}{\gamma_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}},\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\delta_{\mathrm{r}_\ell}}{\delta_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\\ \left[\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\varrho_{\mathrm{r}_\ell}}{\varrho_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}},\frac{1}{1+\left(\frac{1}{x}\sum_{\ell=1}^{x}w_{\mathrm{r}_\ell}\left(\frac{1-\vartheta_{\mathrm{r}_\ell}}{\vartheta_{\mathrm{r}_\ell}}\right)^{\tau}\right)^{\frac{1}{\tau}}}\right]\end{array}\right) \tag{38}$$

$$\bigotimes_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\left(\frac{\overset{x}{\underset{\ell=1}{\oplus}} w_{\mathfrak{r}_\ell}\alpha'_{\mathfrak{r}_\ell}}{x}\right)=\left(\begin{array}{c}\left[\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{\alpha_{\mathfrak{r}_\ell}^{n}}{1-\alpha_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{\beta_{\mathfrak{r}_\ell}^{n}}{1-\beta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\gamma_{\mathfrak{r}_\ell}}{\gamma_{\mathfrak{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\delta_{\mathfrak{r}_\ell}}{\delta_{\mathfrak{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\varrho_{\mathfrak{r}_\ell}}{\varrho_{\mathfrak{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\vartheta_{\mathfrak{r}_\ell}}{\vartheta_{\mathfrak{r}_\ell}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\end{array}\right) \tag{39}$$

$$IVT-SPFDDHM^{(x)}\left(\varphi'_1,\varphi'_2,\dots,\varphi'_{\mathfrak{h}}\right)=\left(\bigotimes_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\left(\frac{\overset{x}{\underset{\ell=1}{\oplus}} w_{\mathfrak{r}_\ell}\varphi'_{\mathfrak{r}_\ell}}{x}\right)\right)^{\frac{1}{C_{\mathfrak{h}}^{x}}}$$

$$=\left(\begin{array}{c}\left[\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^{x}}\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{\alpha_{\mathfrak{r}_\ell}^{n}}{1-\alpha_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^{x}}\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{\beta_{\mathfrak{r}_\ell}^{n}}{1-\beta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^{x}}\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\gamma_{\mathfrak{r}_\ell}^{n}}{\gamma_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^{x}}\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\delta_{\mathfrak{r}_\ell}^{n}}{\delta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^{x}}\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\varrho_{\mathfrak{r}_\ell}^{n}}{\varrho_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}},1-\frac{1}{1+\left[\frac{1}{C_{\mathfrak{h}}^{x}}\sum_{1\le \mathfrak{r}_1<\cdots<\mathfrak{r}_x\le \mathfrak{h}}\frac{x}{\sum_{\ell=1}^{x}w_{\mathfrak{r}_\ell}\left(\frac{1-\vartheta_{\mathfrak{r}_\ell}^{n}}{\vartheta_{\mathfrak{r}_\ell}^{n}}\right)^{\tau}}\right]^{\frac{1}{\tau}}}\right]\end{array}\right) \tag{40}$$

Example 4: Let $\alpha_1' = ([0.6,0.92],[0.2,0.3],[0.3,0.5])$ $\alpha_2' = ([0.77,0.8],[0.11,0.23][0.10,0.11])$, $\alpha_3' = ([0.66,0.73],[0.18,0.25][0.2,0.4])$, and $\alpha_4' = ([0.56,0.82],[0.11,0.23],[0.16,0.22])$ be four IVT-SPFNs and $x = 2, \tau = 3, n = 3$ and $weight = 0.3287, 0.2878, 0.2092, 0.1743$.

$$\frac{\alpha_{\mathfrak{r}_\ell}^n}{1-\alpha_{\mathfrak{r}_\ell}^n} = (0.2842, 0.9531, 1.5706, 0.8137)$$

$$\frac{\gamma_{\mathfrak{r}_\ell}^n}{1-\gamma_{\mathfrak{r}_\ell}^n} = (36.0370, 81.1895, 63.0000, 81.1895)$$

$$\frac{\varrho_{\mathfrak{r}_\ell}^n}{1-\varrho_{\mathfrak{r}_\ell}^n} = (7.0000, 750.3148, 14.6250, 92.9144)$$

$$\frac{1-\beta_{\mathfrak{r}_\ell}^n}{\beta_{\mathfrak{r}_\ell}^n} = (0.2755, 0.8400, 0.4035, 0.2130)$$

$$\frac{1-\delta_{\mathfrak{r}_\ell}^n}{\delta_{\mathfrak{r}_\ell}^n} = (0.0081, 0.0013, 0.005866, 0.001333)$$

$$\frac{1-\vartheta_{\mathfrak{r}_\ell}^n}{\vartheta_{\mathfrak{r}_\ell}^n} = (0.0277, 0.0010, 0.0081, 0.0041)$$

$$IVSFDHM^{(x)}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathfrak{h}}) = \frac{\oplus_{1 \le \mathfrak{r}_1 < \cdots < \mathfrak{r}_x \le \mathfrak{h}} \left(\overset{x}{\underset{\ell=1}{\otimes}} \alpha'_{\mathfrak{r}_\ell} \right)^{\frac{1}{x}}}{C_{\mathfrak{h}}^x}$$

$$= \left(\begin{array}{l}
\left[\frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\frac{1}{0.2740\times(0.2842)^3+0.2564\times(0.9531)^3}+\frac{1}{0.2740\times(0.2842)^3+0.2524\times(1.5706)^3}+\frac{1}{0.2740\times(0.2842)^3+0.2173\times(0.8137)^3} +\frac{1}{0.2564\times(0.9531)^3+0.2524\times(1.5706)^3}+\frac{1}{0.2564\times(0.9531)^3+0.2173\times(0.8137)^3}+\frac{1}{0.2524\times(1.5706)^3+0.2173\times(0.8137)^3}\right)\right)^{\frac{1}{3}}\right)}, \right. \\
\left. 1-1/\left(1+\left(\frac{2}{C_4^2}\times\left(\frac{1}{0.2740\times(36.0370)^3+0.2564\times(81.1895)^3}+\frac{1}{0.2740\times(36.0370)^3+0.2524\times(63.0000)^3}+\frac{1}{0.2740\times(36.0370)^3+0.2173\times(81.1895)^3} +\frac{1}{0.2564\times(81.1895)^3+0.2524\times(63.0000)^3}+\frac{1}{0.2564\times(81.1895)^3+0.2173\times(81.1895)^3}+\frac{1}{0.2524\times(63.0000)^3+0.2173\times(81.1895)^3}\right)\right)^{\frac{1}{3}}\right) \right] \\
\left[\frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\frac{1}{0.2740\times(7.0000)^3+0.2564\times(750.3148)^3}+\frac{1}{0.2740\times(7.0000)^3+0.2524\times(14.6250)^3}+\frac{1}{0.2740\times(7.0000)^3+0.2173\times(92.9144)^3} +\frac{1}{0.2564\times(750.3148)^3+0.2524\times(14.6250)^3}+\frac{1}{0.2564\times(750.3148)^3+0.2173\times(92.9144)^3}+\frac{1}{0.2524\times(14.6250)^3+0.2173\times(92.9144)^3}\right)\right)^{\frac{1}{3}}\right)}, \right. \\
\left. 1-\frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\frac{1}{0.2740\times(0.2755)^3+0.2564\times(0.8400)^3}+\frac{1}{0.2740\times(0.2755)^3+0.2524\times(0.4035)^3}+\frac{1}{0.2740\times(0.2755)^3+0.2173\times(0.2130)^3} +\frac{1}{0.2564\times(0.8400)^3+0.2524\times(0.4035)^3}+\frac{1}{0.2564\times(0.8400)^3+0.2173\times(0.2130)^3}+\frac{1}{0.2524\times(0.4035)^3+0.2173\times(0.2130)^3}\right)\right)^{\frac{1}{3}}\right)} \right] \\
\left[\frac{1}{\left(1+\left(\frac{2}{C_4^2}\times\left(\frac{1}{0.2740\times(0.0081)^3+0.2564\times(0.0013)^3}+\frac{1}{0.2740\times(0.0081)^3+0.2524\times(0.005866)^3}+\frac{1}{0.2740\times(0.0081)^3+0.2173\times(0.001333)^3} +\frac{1}{0.2564\times(0.0013)^3+0.2524\times(0.005866)^3}+\frac{1}{0.2564\times(0.0013)^3+0.2173\times(0.001333)^3}+\frac{1}{0.2524\times(0.005866)^3+0.2173\times(0.00133)^3}\right)\right)^{\frac{1}{3}}\right)}, \right. \\
\left. 1-1/\left(1+\left(\frac{2}{C_4^2}\times\left(\frac{1}{0.2740\times(0.0277)^3+0.2564\times(0.0010)^3}+\frac{1}{0.2740\times(0.0277)^3+0.2524\times(0.0081)^3}+\frac{1}{0.2740\times(0.0277)^3+0.2173\times(0.0041)^3} +\frac{1}{0.2564\times(0.0010)^3+0.2524\times(0.0081)^3}+\frac{1}{0.2564\times(0.0010)^3+0.2173\times(0.0041)^3}+\frac{1}{0.2524\times(0.0081)^3+0.2173\times(0.0041)^3}\right)\right)^{\frac{1}{3}}\right) \right]
\end{array} \right)$$

$$= [0.6559, 0.8127][0.0015, 0.9764][0.0033, 0.9324]$$

Property 9: $(Monotonicity)$ If $\alpha'_\ell = ([\alpha_\ell{}^n, \beta_\ell{}^n], [\gamma_\ell{}^n, \delta_\ell{}^n], [\varrho_\ell{}^n, \vartheta_\ell{}^n])(\ell = 1,2,\dots,\mathfrak{h})$ and $\theta'_\ell = ([r_\ell{}^n, p_\ell{}^n], [m_\ell{}^n, l_\ell{}^n], [s_\ell{}^n, d_\ell{}^n])(\ell = 1,2,\dots,\mathfrak{h})$ are two sets of IVT-SFNs. If $\alpha_\ell{}^n \leq r_\ell{}^n, \gamma_\ell{}^n \leq m_\ell{}^n,\ \varrho_\ell{}^n \leq s_\ell{}^n$ and $\beta_\ell{}^n \geq p_\ell{}^n, \delta_\ell{}^n \geq l_\ell{}^n, \vartheta_\ell{}^n \geq d_\ell{}^n$ apply to all ℓ, then

$$IVT - SPFWDDHM^{(x)}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathfrak{h}}) \leq IVT - SPFWDDHM^{(x)}(\theta'_1, \theta'_2, \dots, \theta'_{\mathfrak{h}})$$

Property 10: $(Boundedness)$ Let $\alpha'_\ell = ([\alpha_\ell{}^n, \beta_\ell{}^n], [\gamma_\ell{}^n, \delta_\ell{}^n], [\varrho_\ell{}^n, \vartheta_\ell{}^n])(\ell = 1,2,\dots,\mathfrak{h})$ be a set of IVT-SPFNs. If $\alpha'_{\mathrm{r}}{}^{+} = \begin{pmatrix} [max_{\mathrm{r}}(\alpha_\ell{}^n), max_{\mathrm{r}}(\gamma_\ell{}^n), max_{\mathrm{r}}(\varrho_\ell{}^n)], \\ [min_{\mathrm{r}}(\beta_\ell{}^n), min_{\mathrm{r}}(\delta_\ell{}^n), min_{\mathrm{r}}(\vartheta_\ell{}^n)] \end{pmatrix}$ and If $\alpha'_{\mathrm{r}}{}^{-} = \begin{pmatrix} [min_{\mathrm{r}}(\alpha_\ell{}^n), min_{\mathrm{r}}(\gamma_\ell{}^n), min_{\mathrm{r}}(\varrho_\ell{}^n)], \\ [max_{\mathrm{r}}(\beta_\ell{}^n), max_{\mathrm{r}}(\delta_\ell{}^n), max_{\mathrm{r}}(\vartheta_\ell{}^n)] \end{pmatrix}$. Then

$$\alpha'^{-} \leq IVT - SPFWDDHM^{(x)}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathfrak{h}}) \leq \alpha'^{+}$$

4. Example and comparison

In this section we present numerical example, compare the result with different parameters and compare the result in prevised aggregation operators.

4.1 Numerical example

In this section we present numerical example based on coverage of COVID and what we think could be used when we enter COVID pandemic.

The media has extensively covered the COVID-19 pandemic, providing information on the spread of the virus, public health measures, vaccination efforts, and related topics. Some might be referring to health insurance coverage related to COVID-19. Many health insurance plans have provided coverage for COVID-19 testing, treatment, and vaccination. Get vaccinated against COVID-19. Vaccination significantly reduces the risk of severe illness and helps prevent the spread of the virus. Practice regular hand washing with soap and water for at least 20 seconds or use hand sanitizer with at least 60% alcohol. Maintain physical distance from others, especially if social distancing measures are recommended in a particular setting. Monitor for COVID-19 symptoms, such as fever, cough, and shortness of breath. Stay home if you feel unwell. Consider regular COVID-19 testing, especially if you have symptoms or have been in contact with someone who tested positive. It is important to stay informed about the latest recommendations from health authorities, as guidelines may evolve based on the current status of the pandemic and the emergence of new variants. Wear masks, especially in crowded or indoor settings. Masks help prevent the transmission of respiratory droplets. Face masks are crucial in preventing the spread of respiratory droplets that may contain the virus. They provide a barrier that helps protect both the wearer and those around them. Properly worn masks, especially those meeting certain standards (such as N95 masks), can significantly reduce the transmission of respiratory particles. Masks are particularly effective when used consistently in combination with other preventive measures.

Types of Marks:

- Surgical Masks B_1: Typically used by healthcare professionals. They provide a physical barrier but vary in effectiveness.
- N95/KN95 Masks B_2: Designed to filter out airborne particles, including viruses. They offer a higher level of protection.
- Cloth Masks B_3: Provide a basic level of protection and are recommended for use in community settings.
- Double Masking B_4: Some health experts recommend wearing two masks (a cloth mask over a surgical mask) to improve the fit and enhance protection.
- Valve Masks B_5: Masks with exhalation valves may not be as effective in preventing the spread of respiratory droplets, and they are often not recommended in certain settings.

Masks should cover the nose and mouth completely. Hands should be clean when putting on or removing masks. Regular replacement or washing of reusable masks is important. The choice of mask may depend on factors like the level of exposure and the surrounding environment. Masks are just one part of a comprehensive approach that includes vaccination, hand hygiene, and social distancing. Follow guidelines provided by health authorities for mask usage in specific settings. Policies on mask wearing may vary based on the prevalence of the virus in a given area. Masks are a valuable tool in reducing the transmission of COVID-19 when used correctly and in conjunction with other preventive measures. It's essential to stay informed about public health recommendations and guidelines in your specific region.

This example consists of five different types of masks. Out of the five antivirus mask types $B_{\mathfrak{r}}(\mathfrak{r}=1,2,3,4,5)$ listed above, we will purchase one antivirus mask. Five applications have been chosen for further review after pre-screening. Based on the following 5 candidates $E_{\mathfrak{r}}\ (\mathfrak{r}=1,2,3,4)$ form four characteristics, E_1 reusability, E_2 quality of raw materials, E_3 filtering efficiency and E_4 leakage rate, which measures how sticky the mask structure is designed to cover the human face. Different weights are used $0.35, 0.35, 0.10\ and\ 0.20$. We attempt to analyse the IVT-SPFN decision matrix by gathering it, as shown in the Table 1.

Table 1. The IVT-SPFN decision matrix

	$\boldsymbol{E_1}$	$\boldsymbol{E_2}$
$\boldsymbol{B_1}$	([0.5,0.92], [0.1,0.3], [0.32,0.5])	([0.67,0.8], [0.15,0.23], [0.10,0.15])
$\boldsymbol{B_2}$	([0.5,0.7], [0.1,0.2], [0.3,0.32])	([0.4,0.9], [0.16,0.22], [0.11,0.16])
$\boldsymbol{B_3}$	([0.67,0.8], [0.2,0.4], [0.23,0.4])	([0.4,0.83], [0.22,0.3], [0.16,0.22])
$\boldsymbol{B_4}$	([0.5,0.9], [0.1,0.3], [0.21,0.31])	([0.44,0.66], [0.31,0.4], [0.11,0.31])
$\boldsymbol{B_5}$	([0.66,0.83], [0.3,0.4], [0.31,0.33])	([0.52,0.56], [0.22,0.32], [0.10,0.22])
	E_3	E_4
$\boldsymbol{B_1}$	([0.55,0.73], [0.15,0.25], [0.18,0.4])	([0.44,0.82], [0.16,0.23], [0.17,0.22])
$\boldsymbol{B_2}$	([0.52,0.82], [0.22,0.35], [0.23,0.35])	([0.56,0.92], [0.12,0.22], [0.22,0.25])
$\boldsymbol{B_3}$	([0.55,0.65], [0.21,0.22], [0.22,0.33])	([0.81,0.91], [0.32,0.46], [0.22,0.35])
$\boldsymbol{B_4}$	([0.56,0.72], [0.11,0.22], [0.21,0.33])	([0.72,0.82], [0.32,0.43], [0.25,0.38])
$\boldsymbol{B_5}$	([0.61,0.78], [0.35,0.52], [0.12,0.22])	([0.68,0.82], [0.22,0.45], [0.19,0.22])

The method created for choosing the top antivirus mask is then applied.

Step 1: According to IVT-SFNs $r_{\mathfrak{r}_\ell}(\mathfrak{r}=1,2,3,4,5, \ell=1,2,3,4)$, we fuse all IVT-SPFNs $r_{\mathfrak{r}_\ell}$ by the IVT-SPFWDHM, IVT-SPFWDDHM operator to have the IVT-SFNs $B_{\mathfrak{r}}(\mathfrak{r}=1,2,3,4,5)$ of the antivirus mask $B_{\mathfrak{r}}$. If $x=2$ then the fused results are presented in the Table 2.

Step 2: Calculate the values of $T^1_{\mathfrak{r}_\ell}, T^2_{\mathfrak{r}_\ell}, T^3_{\mathfrak{r}_\ell}$ based on

$$T=\begin{pmatrix} 1 & 0.9060 & 0.8433 & 0.6933 \\ 1 & 0.7997 & 0.7391 & 0.6365 \\ 1 & 0.8976 & 0.7729 & 0.6117 \\ 1 & 0.9290 & 0.6960 & 0.5781 \\ 1 & 0.9009 & 0.6789 & 0.5671 \end{pmatrix}$$

Step 3: Calculate the weight of the following values in the Table 2.

Table 2. Calculate the weights

0.3932	0.3562	0.0947	0.1558
0.4211	0.3368	0.0889	0.1532
0.4052	0.3637	0.0895	0.1416
0.4068	0.3779	0.0809	0.1344
0.4134	0.3724	0.0802	0.1340

Use IVT-SPFWDDHM and IVT-SPFWDDHM operator to aggregate, as shown in the Table 3.

Table 3. The results received by the IVT-SPFWDHM and IVT-SPFWDDHM operator

	IVT-SPFWDHM
$\boldsymbol{B_1}$	([0.2357,0.5711], [0.0091,0.9949], [0.0084,0.9896])
$\boldsymbol{B_2}$	([0.1950,0.5443], [0.0079,0.9955], [0.0114,0.9794])
$\boldsymbol{B_3}$	([0.3271,0.5371], [0.0261,0.9821], [0.0185,0.9842])
$\boldsymbol{B_4}$	([0.2603,0.6868], [0.0261,0.9735], [0.0221,0.9848])
$\boldsymbol{B_5}$	([0.3533,0.6724], [0.0475,0.9715], [0.0074,0.9907])
-	IVT-SPFWDDHM
$\boldsymbol{B_1}$	([0.6629,0.9046], [0.0017,0.9742], [0.0029,0.9158])
$\boldsymbol{B_2}$	([0.7521,0.9218], [0.0015,0.9823], [0.0059,0.9660])
$\boldsymbol{B_3}$	([0.6186,0.8828], [0.0073,0.9378], [0.0054,0.9471])
$\boldsymbol{B_4}$	([0.6355,0.9002], [0.0012,0.9381], [0.0049,0.9002])
$\boldsymbol{B_5}$	([0.6211,0.8465], [0.0111,0.8766], [0.0013,0.9775])

Step 4: In the Table 4 are used score values.

Table 4. The score values

Alternatives	IVPFWDHM	IVPFWDDHM
$\boldsymbol{B_1}$	0.0817	0.4463
$\boldsymbol{B_2}$	0.0808	0.4532
$\boldsymbol{B_3}$	0.0964	0.4169
$\boldsymbol{B_4}$	0.1547	0.4428
$\boldsymbol{B_5}$	0.1529	0.4129

Step 5: The table 4 lists the antivirus masks in Table 3 order. Moreover, the antivirus mask are both B_4 and B_2. Because our result IVT-SPFDHM and IVT-SPFDDHM are not the same, then we compare the result based on certain operators shown in the Table 5.

Table 5. The order of the antivirus mask

Methods	Order
IVPFWDHM	$B_4 > B_5 > B_3 > B_1 > B_2$
IVPFWDDHM	$B_2 > B_1 > B_4 > B_3 > B_5$

4.2 Influence analysis

The tables 5 to 6 present the analysis of the results and illustrate how the ordering is affected by varying the parameters for the IVT-SPFWDHM and IVT-SPFWDDHM operators which is shown in the Table 6.

Table 6. The order of the operator in IVT-SPFWDHM with different parameters

Parameters	$\boldsymbol{Order}$
$\boldsymbol{X = 1}$	$B_2 > B_1 > B_4 > B_3 > B_5$
$\boldsymbol{X = 2}$	$B_4 > B_5 > B_3 > B_2 > B_1$
$\boldsymbol{X = 3}$	$B_4 > B_1 > B_5 > B_2 > B_3$
$\mathbf{X = 4}$	$B_4 > B_1 > B_2 > B_3 > B_5$

This result shows that the B_4 is the best result because B_4 is the same in more comparisons which is shown in the Table 7.

Table 7. IVT-SPFWDHM results with different parameters

Methods	Order
IVPFWDHM	$B_4 > B_5 > B_3 > B_2 > B_1$

The IVT-SPFWDHM graphical result in the Figure 1 has the same result with different parameters.

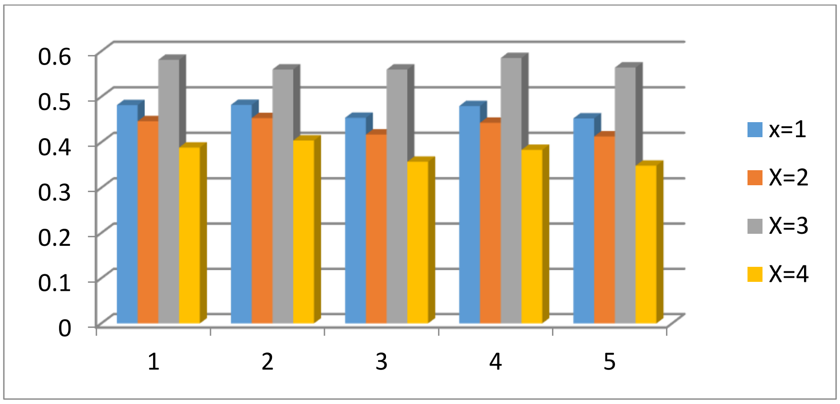

Figure 1. Sensitivity analysis of IVT-SPFWDHM

The analysis of results and illustration how the ordering is affected by varying the parameters for the IVT-SPFWDDHM operators is shown in the Table 8.

Table 8. The order of the operator in IVPFWDDHM with different parameters

Parameters	**Order**
$\mathbf{X = 1}$	$B_5 > B_4 > B_3 > B_1 > B_2$
$\mathbf{X = 2}$	$B_2 > B_1 > B_4 > B_3 > B_5$
$\mathbf{X = 3}$	$B_2 > B_4 > B_3 > B_1 > B_5$
$\mathbf{X = 4}$	$B_5 > B_4 > B_3 > B_2 > B_1$

This result shows that the B_2 is the best result because B_2 is the same in more comparisons which is shown in the Figure 2.

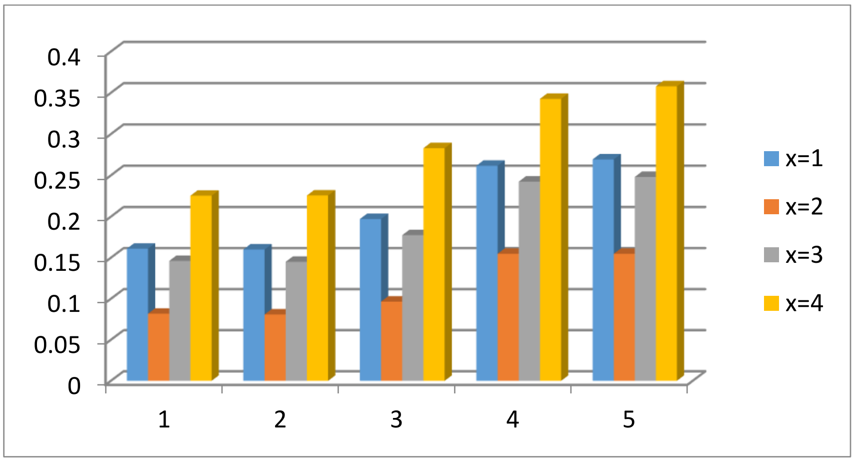

Figure 2. Sensitivity analysis of IVT-SPFWDDHM

4.3 Comparative analysis

We prepared comparisons based on different approaches published in Sarfraz et al. (2022), Garg et al. (2023) and Ullah et al. (2021) . The result are presented in the Table 9.

Table 9. The compared results

Methods	Order
$IVTSPWDHM$	$B_4 > B_4 > B_3 > B_1 > B_2$
$IVTSPWDDHM$	$B_2 > B_1 > B_4 > B_3 > B_5$
Sarfraz et al. (2022)	$B_4 > B_4 > B_3 > B_1 > B_2$
Sarfraz et al. (2022)	$B_2 > B_1 > B_4 > B_3 > B_5$
Garg et al. (2023)	$B_4 > B_4 > B_3 > B_1 > B_2$
Garg et al. (2023)	$B_2 > B_1 > B_4 > B_3 > B_5$
Ullah et al. (2021)	$B_4 > B_4 > B_3 > B_1 > B_2$
Ullah et al. (2021)	$B_2 > B_1 > B_4 > B_3 > B_5$

We compare the results in different operator to find the ranking of the result in the Table 8 and then show the result clearly. The N95/KN95 masks B_2 according to the result is IVT-SFDWHM and Double Masking according to IVT-SFDDWHM it is B_5. The Figure 3 shows the result of prioritized HM in different operators.

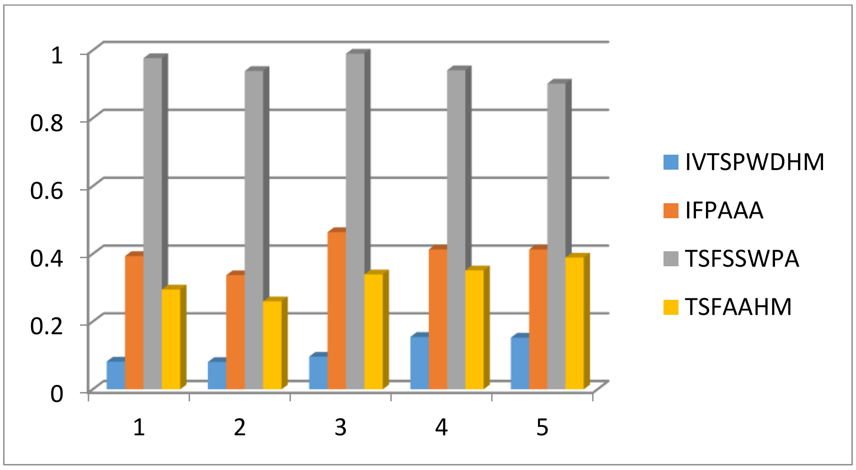

Figure 3. IVT-SFDWHM comparison with different operators

The Figure 4 shows the result of prioritized DHM in different operators.

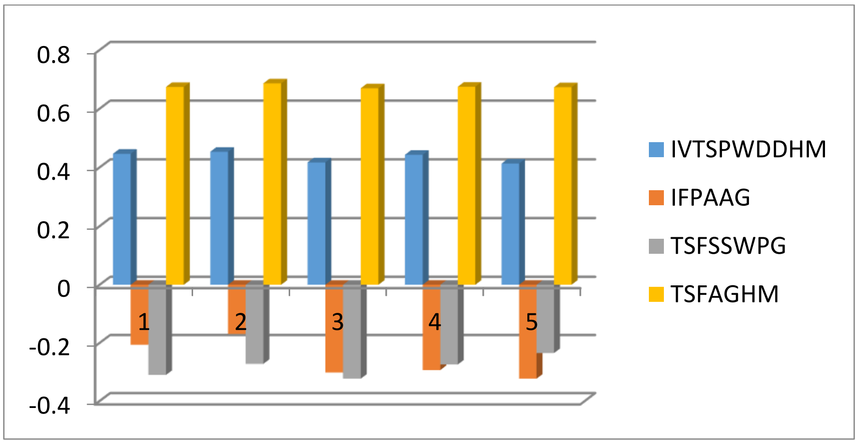

Figure 4. IVT-SFDDWHM comparison with different operators

Form the above, comparative study shows that N95/KN95 masks B_2 and Double Masking B_5 is the best selection in terms of types of masks.

5. Conclusions

The utilization of IVT-SPFDHM and IVT-SPFDDHM operators in group antivirus mask analysis has demonstrated its effectiveness in handling uncertainty and prioritized fuzzy data. The applications of IVT-SPFDHM operator in various decision-making scenarios underline its adaptability and robustness, making it a valuable asset for addressing complex real-world problems in group decision-making contexts. We provided a numerical example of antivirus mask for fighting COVID carried out using a MADM algorithm in order to demonstrate the effects of the proposed IVT-SPFDHM and IVT-SPFDDM operators. The prioritization relationship between criteria and decision-makers is taken into account by the suggested MADM method, which gives our method a broad range of practical application possibilities. Additionally, we analysed the results and compared them to those of other AOs using the prioritization idea. We discovered that the outcomes are consistent and similar. Furthermore, we presented how these prioritized AOs may affect interval valued T-SFs. Additionally, we plan to expand the work in order to include complex spherical fuzzy sets, and complex IVT-SFPDHM fuzzy sets.

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